Kattabrig problem for nonlinear equations.

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Annotation: For nonlinear equations, the Kattabrig problem has been used in some boundary value problems for third-order equations. Such equations are called multiple equations. For equations of the third order multiple characteristics in a finite field, the problem of a nonlinear equation in a boundary value problem is posed and the Green function is constructed for this problem, and the existence and uniqueness of the solution of the problem is indicated. It is also important to show the existence and uniqueness of the solution of boundary value problems in infinite fields for the equation of the third order multiple characteristics.

Keywords: differential equation, special product, Kattabrig problem, Green's formula, boundary value problem.

In this chapter, the limited area has the following appearance

$$\frac{\partial^3 u}{\partial x^3} - \beta(x) \frac{\partial u}{\partial y} = F\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}\right),\,$$

some boundary value problems are considered for third-order equations.

Such equations are called multiple equations

§ 2.1. Problem Setting and the Green's Function.

$$D\{0 < x < 1, 0 < y \le 1\}$$

$$L(u) \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial y} = F(x, y, u, u_x)$$
 (1.1)

for the equation we consider the following boundary value problem.

K issue

For equation (1.1) in the closed area \overline{D} continuous with the product U_x and satisfying the following boundary conditions

$$u(0,y) = \varphi_1(y), \qquad u_x(0,y) = \varphi_2(y), u(1,y) = \varphi_3(y), \qquad 0 \le y \le 1 u(x,0) = \varphi_0(x), \qquad 0 \le x \le 1$$
 (1.2)

find a regular solution.

In this φ_1 , φ_2 , φ_3 given continuous functions; $\varphi_0(x)$ continuous

with its product and $\varphi_1(0) = \varphi_0(0)$, $\varphi_3(0) = \varphi_0(1)$, $\varphi_2(0) = \varphi_0(0)$ (1.1)- (1.2)

the problem is studied in Kattabri for a linear equation.

Green function for K issue.

$$L(u) = \frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial y} = 0 \tag{1.3}$$

check the equation.

The following is true

$$\varphi L(\psi) - \psi M(\varphi) = \frac{\partial}{\partial \xi} \left(\varphi \psi_{\xi\xi} - \varphi_{\xi} \psi_{\xi} + \varphi_{\xi\xi} \psi \right) - \frac{\partial}{\partial \eta} (\varphi \psi),$$

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In this $M \equiv \frac{\partial}{\partial y} - \frac{\partial^3}{\partial x^3}$ - L operator with operator and ψ - smooth functions. We integrate the given identity in field D and obtain the following result

$$\iint\limits_{D} [\varphi L(\psi) - \psi M(\varphi)] d\xi d\eta = \int\limits_{\Gamma} (\varphi \psi_{\xi\xi} - \varphi_{\xi} \psi_{\xi} + \varphi_{\xi\xi} \psi) d\eta + (\varphi \psi) d\xi, (1.4)$$

where the integral D on the right is taken along the entire boundary of the field. It is known that the fundamental solution of [(1.3) has the following form

$$U(x, y; \xi, \eta) = \begin{cases} \frac{1}{(y - \eta)^{1/3}} f\left(\frac{x - \xi}{(y - \eta)^{1/3}}\right), & y > \eta, \\ 0, & y \le \eta, \end{cases}$$

$$V(x, y; \xi, \eta) = \begin{cases} \frac{1}{(y - \eta)^{1/3}} \varphi\left(\frac{x - \xi}{(y - \eta)^{1/3}}\right), & y > \eta, \\ 0, & y \le \eta, \end{cases}$$

Inthis

$$f(t) = \frac{\pi\sqrt{t}}{3\sqrt{3}} \left[I_{\frac{1}{3}} \left(\frac{2}{3\sqrt{3}} t^{3/2} \right) + I_{-\frac{1}{3}} \left(\frac{2}{3\sqrt{3}} t^{3/2} \right) \right];$$

$$\varphi(t) = \frac{\pi\sqrt{t}}{3} \left[I_{\frac{1}{3}} \left(\frac{2}{3\sqrt{3}} t^{3/2} \right) - I_{-\frac{1}{3}} \left(\frac{2}{3\sqrt{3}} t^{3/2} \right) \right]; \tag{1.5}$$

 $I_{\nu}(z)$ – Bessel function f(t) va $\varphi(t)$ functions They are called curved functions [80] $z''(t) + \frac{t}{3}z(t) = 0 \tag{1.6}$

satisfies the equation

$$U(x, y; \xi, \eta)$$
 and $V(x, y; \xi, \eta)$

The following estimates are valid for functions In this

$$\begin{vmatrix} \frac{\partial^{i+j}U(x,y;\xi,\eta)}{\partial x^{i}\partial y^{j}} \\ \left| \frac{\partial^{i+j}V(x,y;\xi,\eta)}{\partial x^{i}\partial y^{j}} \right| \end{vmatrix} < C \frac{|x-\xi|^{\frac{2i+6j-1}{4}}}{|y-\eta|^{\frac{2i+6j+1}{4}}}$$
herein
$$\frac{x-\xi}{(y-\eta)^{1/3}} \to +\infty, \quad i+j \ge 1, \quad C-const > 0$$

$$\left| \frac{\partial^{i+j}U(x,y;\xi,\eta)}{\partial x^{i}\partial y^{j}} \right| < \frac{C_{1}}{|y-\eta|^{\frac{i+3j+1}{3}}} \exp\left(-C_{2} \frac{|x-\xi|^{3/2}}{|y-\eta|^{1/2}}\right)$$
herein
$$\frac{x-\xi}{(y-\eta)^{1/3}} \to -\infty, \quad i+j \ge 1, \quad C_{1}, C_{2}-const > 0$$

Now in formula φ (1.4) and ψ we get the functions U(x,y) and $U(x,y,\xi,\eta)$ and instead of the functions ph and ps, respectively. U(x,y) (1.3) is an arbitrary regular solution of equation. Let D^{ε} be defined by the inequalities $\{0<\xi<1,\ 0<<\eta\leq y-\varepsilon\}$ where $\varepsilon>0$ — is a small number. In this case (1.4) the identity is written as follows.

$$0 = \int_{0}^{y} (U_{\xi\xi}u - U_{\xi}u_{\xi} + Uu_{\xi\xi})|_{\xi=1} d\eta - \int_{0}^{y-\varepsilon} (U_{\xi\xi}u - U_{\xi}u_{\xi} + Uu_{\xi\xi})|_{\xi=1} d\eta +$$

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$$+\int_{0}^{1} Uu|_{\eta=y-\varepsilon} d\xi - \int_{0}^{1} Uu|_{\eta=0} d\xi.$$

We integrate e to 0 and consider equation [87]

$$\lim_{\varepsilon \to 0} \int_0^1 U(x, y; \xi, y - \varepsilon) u(\xi, y - \varepsilon) d\xi = \pi u(x, y),$$

hence

$$\pi u(x,y) = \int_{0}^{y} \left(U_{\xi\xi} u - U_{\xi} u_{\xi} + U u_{\xi\xi} \right) |_{\xi=0} d\eta - \int_{0}^{y} \left(U_{\xi\xi} u - U_{\xi} u_{\xi} + U u_{\xi\xi} \right) |_{\xi=1} d\eta +$$

$$+ \int_{0}^{1} U(x, y; \xi, 0) u(\xi, 0) d\xi.$$
 (1.8)

now

$$W(x, y; \xi, \eta)$$

$$M(v) \equiv \frac{\partial v}{\partial \eta} - \frac{\partial^3 v}{\partial \xi^3} = 0$$
(1.9)

Let U(x,y) be a regular solution of equation and be an arbitrary regular solution of equation (1.3). In this case (1.4) we say $\varphi = W$, $\psi = U$

$$0 = \int_{0}^{y} (u_{\xi\xi}W - W_{\xi}u_{\xi} + uW_{\xi\xi})|_{\xi=1} d\eta - \int_{0}^{y} (u_{\xi\xi}W - W_{\xi}u_{\xi} + uW_{\xi\xi})|_{\xi=0} d\eta + \int_{0}^{1} Wu|_{\eta=y} d\xi - \int_{0}^{1} Wu|_{\eta=0} d\xi.$$

$$(1.10)$$

We find from (1.8) and (1.10)

$$\pi u(x,y) = \int_{0}^{y} \left[\left(u_{\xi\xi}(W - U) + u_{\xi}(-W + U)_{\xi} + u(W - U)_{\xi\xi} \right) \right] |_{\xi=0} d\eta + \int_{0}^{y} \left[\left(u_{\xi\xi}(-W + U) + u_{\xi}(W - U)_{\xi} + u(-W + U)_{\xi\xi} \right) \right] |_{\xi=0} d\eta + U(-W + U)_{\xi\xi}$$

$$+\int_{0}^{1}(-W+U)u|_{\eta=0}\,d\xi+\int_{0}^{1}Wu|_{\eta=y}\,d\xi.$$
 (1.11)

If $W(x, y; \xi, \eta)$ is a regular solution of (1.9) and satisfies the following conditions $W(x, y; \xi, \eta)|_{\xi=1} = U(x, y; \xi, \eta)|_{\xi=1}$, $W_{\varepsilon=0} = U|_{\xi=0}$, $W_{\xi}|_{\xi=1} = U_{\xi}|_{\xi=1}$, $W|_{\eta=y} = 0$, (1.12)

from (1.11)

$$\pi u(x,y) = \int_{0}^{y} G_{\xi\xi}(x,y;o,\eta)u(0,\eta)d\eta - \int_{0}^{y} G_{\xi}(x,y;o,\eta)u_{\xi}(0,\eta)d\eta -$$

$$-\int_{0}^{y} G_{\xi\xi}(x,y;1,\eta)u(1,\eta)d\eta + \int_{0}^{1} G(x,y;\xi,0)u(\xi,0)d\xi, \tag{1.13}$$

arises in this

$$G(x, y; \xi, \eta) = U(x, y; \xi, \eta) - W(x, y; \xi, \eta)$$

(1.13) formula K. gives the solution of the problem. However, the existence of the function $W(x, y; \xi, \eta)$ must be proved, and this function must satisfy the conditions (1.9) and (1.12). We will later prove the existence of the function $W(x, y; \xi, \eta)$ and now we will show some properties of the function $G(x, y; \xi, \eta)$

$$\pi \bar{u}(\xi,\eta) = \int_{\eta}^{1} (U_{xx}\bar{u} - U_{x}\bar{u}_{x} + U\bar{u}_{xx})|_{x=1} dy - \int_{\eta}^{1} (U_{xx}\bar{u} - U_{x}\bar{u}_{x} + U\bar{u}_{xx})|_{x=0} dy +$$

$$+ \int_{0}^{1} U(x,1;\xi,\eta) \bar{u}(x,1) dx, \tag{1.14}$$

This is similar to formula (1.8) and follows

$$0 = \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=1}dy - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx - \int_{\eta}^{1} (\overline{u}_{xx}\overline{W} - \overline{u}_{x}\overline{W}_{x} + \overline{u}\overline{W}_{xx})|_{x=0}dy + \int_{0}^{1} \overline{W}\overline{u}|_{y=1}dx + \int_{0}^{1} \overline{$$

$$-\int_{0}^{1} \overline{W}\overline{u}|_{y=\eta} dx, \tag{1.15}$$

the formula is similar to (1.10). Using formulas (1.14) and (1.15) and the properties of the function $\overline{W}(x,y;\xi,\eta)$ we obtain the following result

$$\pi \bar{u}(\xi,\eta) = \int_{\eta}^{1} \bar{G}_{xx}(1,y;\xi,\eta) \bar{u}(1,y) dy - \int_{\eta}^{1} \bar{G}_{xx}(0,y;\xi,\eta) \bar{u}(0,y) dy -$$

$$-\int_{\eta}^{1} \bar{G}_{x}(1,y;\xi,\eta) \bar{u}_{x}(1,y) dy + \int_{0}^{1} \bar{G}(x,1;\xi,\eta) \bar{u}(1,x) dx, \tag{1.16}$$

herein

$$\bar{G}(x, y; \xi, \eta) = U(x, y; \xi, \eta) - \overline{W}(x, y; \xi, \eta).$$

Obviously, that's it

$$\overline{G}|_{x=0} = \overline{G}_x|_{x=0} = \overline{G}|_{x=1} = \overline{G}|_{y=\eta} = 0.$$
 (1.17)

conditions are met.

 $\eta < \theta < y$ let it be. $\{0 < x < 1, \ \theta < y \le 1\}$ in the field(1.13)) and as a regular solution $G(x,y;\xi,\eta)$ If we take the function, we get the following result

$$\pi \bar{G}(x,y;\xi,\eta) = \int_{0}^{1} G(x,y;x',\theta) \bar{G}(x',\theta;\xi,\eta) dx', \qquad (1.18)$$

because the remaining integrals (1.17) become zero according to the condition.

Now we apply (1.16) to the field $\{0 < \xi < 1, 0 < \eta < \theta\}$ taking $G(x, y; \xi, \eta)$ as a regular solution

$$\pi G(x, y; \xi, \eta) = \int_{0}^{1} \bar{G}(x', \theta; \xi, \eta) G(x, y; x', \theta) dx'$$
 (1.19)

we come to. The remaining integrals are as follows

$$G|_{\xi=1} = G_{\xi}|_{\xi=1} = G_{\xi=0} = G|_{\eta=y} = 0$$
(1.20)

becomes zero according to the condition.

If we compare (1.18) and (1.19)

$$G(x, y; \xi, \eta) = \bar{G}(x, y; \xi, \eta)$$

will be clear. From this $G(x, y; \xi, \eta)$ according to the condition of our function (1.17) (x, y) satisfies on, (1.20) satisfies on condition according to the condition. We call this function the Green function for the K. problem. Not homosexual L(u) = f(x, y) (1.21)

Let the equation be given. In this case to the right of (1.13)

$$-\iint\limits_{D}G(x,y;\xi,\eta)f(\xi,\eta)d\xi d\eta$$

limit This means that the heterogeneous solution of problem K (1.21) has the following form

$$u(x,y) = -\frac{1}{\pi} \iint_{D} G(x,y;\xi,\eta) f(\xi,\eta) d\xi d\eta.$$

 $G(x, y; \xi, \eta)$ satisfying the conditions (1.12) to prove that the function exists $W(x, y; \xi, \eta)$ it is sufficient to prove the existence of a function. We look for the solution of the problem (1.9), (1.12) in the following form

$$W(x, y; \xi, \eta) = \int_{\eta}^{y} U_{x}(1, \tau; \xi, \eta) \alpha_{1}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta) \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} V_{x}(1, \tau; \xi, \eta) \alpha_{3}(x, y, \tau) d\tau,$$
(1.22)

Herein $U(x,y;\xi,\eta)$ and $V(x,y;\xi,\eta)$ — the above functions $\alpha_i(x,y,\tau)$ (i=1,2,3)

currently unknown features. (1.12) according to boundary conditions

$$U(x, y; 0, \eta) = \int_{\eta}^{y} U_{x}(1, \tau; 0, \eta) \alpha_{1}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; \xi, \eta)|_{\xi=0} \alpha_{2}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0,$$

$$+ \int_{\eta}^{y} V_{x}(1,\tau;0,\eta) \,\alpha_{3}(x,y,\tau) d\tau, \tag{1.23}$$

$$U(x, y; 1, \eta) = \int_{\eta}^{y} U_{x}(1, \tau; 1, \eta) \alpha_{1}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx}(0, \tau; 1, \eta) \alpha_{2}(x, y, \tau) d\tau +$$

$$+ \int_{\eta}^{y} V_{x}(1,\tau;1,\eta) \,\alpha_{3}(x,y,\tau) d\tau, \tag{1.24}$$

$$U_{\xi}(x, y; 1, \eta) = \int_{\eta}^{y} U_{x\xi}(1, \tau; \xi, \eta)|_{\xi=1} \alpha_{1}(x, y, \tau) d\tau + \int_{\eta}^{y} U_{xx\xi}(0, \tau; 1, \eta) \alpha_{2}(x, y, \tau) d\tau +$$

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$$+ \int_{\eta}^{y} V_{x\xi}(1,\tau;\xi,\eta)|_{\xi=1} \alpha_{3}(x,y,\tau)d\tau.$$
 (1.25)

This system α_i is a system of integral equations of the type Voltaire with respect to unknown functions. We show that the system of equations (1.23), (1.24), (1.25) has solutions with the following properties

$$\alpha_2 \in L_2(0, y), \qquad \alpha_1, \qquad \alpha_3 \in C(0, y)$$

(1.23) and

$$U(x, y; 0, \eta) = \int_{\eta}^{y} \frac{1}{(\tau - \eta)^{2/3}} f'\left(\frac{1}{(\tau - \eta)^{1/3}}\right) \alpha_{1}(x, y, \tau) d\tau +$$

$$+ \int_{\eta}^{y} U_{xx}(0,\tau;\xi,\eta)|_{\xi=0} \alpha_{2}(x,y,\tau)d\tau + \int_{\eta}^{y} \frac{1}{(\tau-\eta)^{2/3}} \varphi'\left(\frac{1}{(\tau-\eta)^{1/3}}\right) \times \alpha_{3}(x,y,\tau)d\tau.$$

Let's look at the second participant

$$I = \int_{\eta}^{y} U_{xx}(0,\tau;\xi,\eta) \,\alpha_{2}(x,y,\tau)d\tau = \frac{1}{3} \int_{\eta}^{y} \frac{\xi}{(\tau-\eta)^{4/3}} f\left(\frac{-\xi}{(\tau-\eta)^{1/3}}\right) \times \alpha_{2}(x,y,\tau)d\tau.$$

Here we used the function f(t) to satisfy equation (1.6).

In this regard

$$\lim_{\xi \to 0} I = \frac{\pi}{3} \alpha_2(x, y, \eta), \text{d} \text{ the following}$$

$$U(x, y; 0, \eta) = \int_{\eta}^{y} \frac{1}{(\tau - \eta)^{2/3}} f'\left(\frac{1}{(\tau - \eta)^{1/3}}\right) \alpha_{1}(x, y, \tau) d\tau + \frac{\pi}{3} \alpha_{2}(x, y, \eta) + \int_{\eta}^{y} \frac{1}{(\tau - \eta)^{2/3}} \varphi'\left(\frac{1}{(\tau - \eta)^{1/3}}\right) \alpha_{3}(x, y, \tau) d\tau.$$
(1.26)

we create.

Now consider equation (1.24)

$$U(x, y; 1, \eta) = f'(0) \int_{\eta}^{y} \frac{\alpha_{1}(x, y, \tau) d\tau}{(\tau - \eta)^{2/3}} + \frac{1}{3} \int_{\eta}^{y} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{1/3}}\right) \times \times \alpha_{2}(x, y, \tau) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{1/3}}\right) \times \alpha_{2}(x, y, \tau) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{1/3}}\right) \times \alpha_{2}(x, y, \tau) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{1/3}}\right) \times \alpha_{2}(x, y, \tau) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{1/3}}\right) \times \alpha_{2}(x, y, \tau) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{1/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{4/3}}\right) d\tau + \frac{1}{3} \int_{\eta}^{\eta} \frac{1}{(\tau - \eta)^{4/3}} d\tau + \frac{1}{3} \int_{\eta}^{\eta$$

$$+\varphi'(0)\int_{\eta}^{y}\frac{\alpha_{3}(x,y,\tau)d\tau}{\left(\tau-\eta\right)^{2/3}}.$$

We multiply both sides of this equation by $(z - \eta)^{-\frac{1}{3}} d\eta$ (z – (z-parameter) and integrate from y to z. Differentiate by z again

$$\int_{y}^{z} \frac{U_{\eta}(x, y; 1, \eta)}{(z - \eta)^{1/3}} d\eta = -f'(0) \frac{2\pi}{\sqrt{3}} \alpha_{1}(x, y, z) +$$

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$$+ \int_{y}^{z} \alpha_{2}(x, y, \tau) d\tau \int_{y}^{z} \frac{1}{(z - \eta)^{1/3}} \frac{\partial}{\partial \eta} \left[\frac{1}{(\tau - \eta)^{4/3}} f\left(\frac{-1}{(\tau - \eta)^{1/3}}\right) \right] d\eta - \varphi'(0) \frac{2\pi}{\sqrt{3}} \alpha_{3}(x, y, z).$$
(1.27)

we form a relationship.

Consider Equation (1.25):

$$U_{\xi}(x,y;1,\eta) = \frac{1}{3} \int_{\eta}^{y} \frac{1-\xi}{(\tau-\eta)^{4/3}} f\left(\frac{1-\xi}{(\tau-\eta)^{1/3}}\right) \Big|_{\xi=1} \alpha_{1}(x,y,\tau) d\tau + \int_{\eta}^{y} \frac{1}{(\tau-\eta)^{4/3}} f'''\left(\frac{-1}{(\tau-\eta)^{1/3}}\right) \alpha_{2}(x,y,\tau) d\tau + \int_{\eta}^{y} \frac{1-\xi}{(\tau-\eta)^{4/3}} \varphi\left(\frac{1-\xi}{(\tau-\eta)^{1/3}}\right) \Big|_{\xi=1} \alpha_{3}(x,y,\tau) d\tau = \frac{2}{3} \pi \alpha_{1}(x,y,\eta) + \int_{\eta}^{y} \frac{1}{(\tau-\eta)^{4/3}} f'''\left(\frac{-1}{(\tau-\eta)^{1/3}}\right) \alpha_{2}(x,y,\tau) d\tau.$$
 (1.28)

In doing so, we took into account the following relationships, which are easily found from (1.5)

$$\int_{0}^{\infty} \varphi(t)dt = 0, \quad \int_{0}^{\infty} f(t)dt = \frac{2}{3}\pi, \quad \int_{-\infty}^{0} f(t)dt = \frac{\pi}{3}.$$

We enter the following definitions:

$$\omega_1(x,y,\eta) = U(x,y;0,\eta), \quad \omega_2(x,y,\eta) = \int_{y}^{\eta} \frac{U_z(x,y;1,z)}{(\eta-z)^{1/3}} dz,$$

$$\omega_3(x, y, \eta) = U_{\xi}(x, y; 1, \eta), \quad K_{11}(\tau, \eta) = \frac{1}{(\tau - \eta)^{2/3}} f'\left(\frac{1}{(\tau - \eta)^{1/3}}\right),$$

$$K_{31}(\tau,\eta) = \frac{1}{(\tau-\eta)^{2/3}} \varphi' \left(\frac{1}{(\tau-\eta)^{1/3}}\right),$$

$$K_{22}(\tau,\eta) = \int_{\tau}^{\eta} \frac{1}{(\eta-z)^{1/3}} \frac{\partial}{\partial z} \left[\frac{1}{(\tau-z)^{4/3}} f\left(\frac{-1}{(\tau-z)^{1/3}}\right) \right] dz,$$

$$K_{32}(\tau,\eta) = \frac{1}{(\tau-\eta)^{4/3}} f'''\left(\frac{1}{(\tau-\eta)^{1/3}}\right).$$

In this case, the system (1.26) - (1.28) has the following form

$$\omega_1(x, y, \eta) = d\tau + \frac{\pi}{3}\alpha_2(x, y, \eta) + \int_{\eta}^{y} K_{31}(\tau, \eta) \alpha_3(x, y, \tau) d\tau,$$
 (1.29)

$$\omega_2(x, y, \eta) = -f'(0) \frac{2\pi}{\sqrt{3}} \alpha_1(x, y, \eta) + \int_{\eta}^{y} K_{23}(\tau, \eta) \alpha_2(x, y, \tau) d\tau - \varphi'(0) \frac{2\pi}{\sqrt{3}} \alpha_3(x, y, \eta),$$

$$\omega_3(x,y,\eta) = \frac{2\pi}{3}\alpha_1(x,y,\eta) + \int_{\eta}^{y} K_{32}(\tau,\eta)\alpha_2(x,y,\tau)d\tau,$$

Herein $\omega_1 \in L_2(0,y)$, $\omega_2, \omega_3 \in C(0,y)$ and $K_{ij}(\tau,y)$ integrable (1.29) from the system α_1 and α_3 if we lose

$$h(x, y, \eta) = \alpha_2(x, y, \eta) + \int_{\eta}^{y} K^*(\xi, \eta) \,\alpha_2(x, y, \xi) d\xi,$$
In this
$$h(x, y, \eta) = \frac{3}{\pi} \omega_1(x, y, \eta) -$$
(1.30)

$$-\frac{9}{4\pi^{2}} \int_{\eta}^{y} \left[K_{11}(\tau, \eta)\omega_{3}(x, y, \tau) - \frac{1}{\varphi'(0)\sqrt{3}} K_{31}(\tau, \eta)\omega_{2}(x, y, \tau) - \frac{f'(0)}{\varphi'(0)} K_{31}(\tau, \eta)\omega_{3}(x, y, \tau) \right] d\tau;$$
(1.31)

$$K^{*}(\xi,\eta) = \int_{\xi}^{y} \left[\frac{gf'(0)}{\varphi'(0)\pi^{2}} K_{31}(\tau,\eta) K_{32}(\xi,\eta) - \frac{3\sqrt{3}}{\varphi'(0)2\pi^{2}} K_{31}(\tau,\eta) K_{22}(\xi,\eta) - \frac{9}{2\pi^{2}} K_{11}(\tau,\eta) K_{32}(\xi,\eta) \right] d\tau.$$
(1.32)

(1.31) and (1.32) are shown $h(x, y, \eta) \in L_2(0, y)$, and $K^*(\xi, \eta)$

the function has a weakness class. It follows that Equation (1.30) has a solution for class L_2. Substituting this solution into (1.29) gives $\alpha_1, \alpha_3 \in C(0, y)$.

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