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A(z)-Application of Ananlytic function Jordan's Lemma

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Annotation. This article dedicated to notion of $A(z)$-analytic function and Jordan's lemma. Jordan's lemma is proved for $A(z)$-analytic function.

Key Words. $A(z)$ -analytic function, antianalytic function, Jordan's lemma.

Consider, $D-$ field □ $\cong$ □ ^{2} depicted in complex plane . If $z = x + iy$, then

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} + \frac{1}{i}\cdot\frac{\partial}{\partial y}\right) \qquad \frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} - \frac{1}{i}\cdot\frac{\partial}{\partial y}\right)$$

$A(z) \in C(D)$ function for us

$$D_A = \frac{\partial}{\partial z} - \overline{A(z)}\cdot\frac{\partial}{\partial \overline{z}}\,, \quad \overline{D_A} = \frac{\partial}{\partial \overline{z}} - A(z)\cdot\frac{\partial}{\partial z}$$

We can write down this.

Let the function $D \subset$ □ $A(z)$ -antianalytic function and $\left(L(a,r) = \left\{|\psi(z,a)| = \left|z - a + \int\limits_{\gamma(a,z)} A(\tau)d\overline{\tau}\right| < r\right\}\right) \subset\subset D$ collection is compact in region D .

1-definition. $f(z) \in C^1(D)$ *function called A-analytic in region* $D-$ *If,* $\forall z \in D$ *points equal to this*
$\overline{D_A} f(z) = 0$.

Jordan's lemma

Let the antianalytic function $D \subset$ □ da $A(z)$ - then , $A = const$ and $|A| < 1$.

Lemma (anolog of Jordan's Lemma). Except for isolated special points $D = \{z \in$ □ $: \operatorname{Im}(z + A\overline{z}) \geq 0\}$ at all points in the set of $f(z)$ consider $A(z)$ -analytic function. $\gamma_R = \{z \in$ □ $: z + A\overline{z} = Re^{i\varphi},\ 0 \leq \varphi \leq \pi\}$ on the curve $M(R) = \max\limits_{\gamma_R}|f(z)| \underset{R\to\infty}{\to} 0$. then, for number $\forall \lambda > 0$

$$\lim_{R\to\infty} \int\limits_{\gamma_R} f(z)e^{i\lambda(z + A\overline{z})}(dz + Ad\overline{z}) = 0\,.$$

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As proof of this . let this equal $\gamma_R = \gamma'_R \cup \gamma''_R$ then, $\gamma'_R = \left\{ z \in C : z + A\bar{z} = Re^{i\varphi},\ 0 \le \varphi \le \frac{\pi}{2} \right\}$.

So that $\varphi \in \left[0, \frac{\pi}{2} \right]$ because of this Jordan's equity is proper $\sin\varphi \ge \frac{2}{\pi}\varphi$. Using this

$$\left| e^{i\lambda(z+A\bar{z})} \right| = \left| e^{i\lambda R(\cos\varphi + i\sin\varphi)} \right| = \left| e^{i\lambda R\cos\varphi - \lambda R\sin\varphi} \right| =$$

$$= \left| e^{i\lambda R\cos\varphi} \cdot e^{-\lambda R\sin\varphi} \right| = \left| e^{i\lambda R\cos\varphi} \right| \cdot \left| e^{-\lambda R\sin\varphi} \right| =$$

$$= \left| \cos(\lambda R\cos\varphi) + i\sin(\lambda R\cos\varphi) \right| \cdot \left| e^{-\lambda R\sin\varphi} \right| =$$

$$= \left| e^{-\lambda R\sin\varphi} \right| \le e^{-\lambda R\frac{2}{\pi}\varphi}$$

We can get this inequality. If $R \to \infty$ then,

$$\left| \int_{\gamma_R} e^{i\lambda(z+A\bar{z})} f(z)(dz + Ad\bar{z}) \right| \le \int_{\gamma_R} \left| e^{i\lambda(z+A\bar{z})} \right| \cdot \left| f(z) \right| \cdot \left| dz + Ad\bar{z} \right| \le$$

$$\left| \begin{matrix} z + A\bar{z} = Re^{i\varphi},\ dz + Ad\bar{z} = iRe^{i\varphi}d\varphi,\ \left| dz + Ad\bar{z} \right| = Rd\varphi, \\ 0 \le \varphi \le \frac{\pi}{2},\ \ M(R) = \max\limits_{R\to\infty} \left| f(z) \right| \end{matrix} \right| .$$

$$\le \int_0^{\frac{\pi}{2}} M(R) \cdot R \cdot e^{-\lambda R\frac{2}{\pi}\varphi} d\varphi = M(R) \cdot R \int_0^{\frac{\pi}{2}} e^{-\lambda R\frac{2}{\pi}\varphi} d\varphi =$$

$$= -M(R) \cdot R \cdot \frac{\pi}{2\lambda R} e^{-\lambda R\frac{2}{\pi}\varphi} \Big|_0^{\frac{\pi}{2}} = M(R)\frac{\pi}{2\lambda}\left(1 - \frac{1}{e^{\lambda R}}\right) \to 0$$

Consider $\gamma''_R = \gamma_R \setminus \gamma'_R$ marked then $\gamma''_R = \left\{ z \in C : z + A\bar{z} = Re^{i(\pi-\varphi)}, 0 \le (\pi - \varphi) \le \frac{\pi}{2} \right\}$ when $0 \le (\pi - \varphi) \le \frac{\pi}{2}$ therefore $\sin(\pi - \varphi) \ge \frac{2}{\pi}(\pi - \varphi)$ equity is proper. In this case

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$$\left|e^{i\lambda(z+A\bar{z})}\right| = \left|e^{i\lambda R\left(\cos(\pi-\varphi)+i\sin(\pi-\varphi)\right)}\right| = \left|e^{i\lambda R\cos(\pi-\varphi)-\lambda R\sin(\pi-\varphi)}\right| =$$

$$= \left|e^{i\lambda R\cos(\pi-\varphi)}\right| \cdot \left|e^{-\lambda R\sin(\pi-\varphi)}\right| =$$

$$= \left|\cos\left(\lambda R\cos(\pi-\varphi)\right) + i\sin\left(\lambda R\cos(\pi-\varphi)\right)\right| \cdot \left|e^{-\lambda R\sin(\pi-\varphi)}\right| =$$

$$= \left|e^{-\lambda R\sin(\pi-\varphi)}\right|$$

Equity is approperite, we evaluate the given integral when $R \to \infty$ is big enough·

$$\left|\int_{\gamma_R''} e^{i\lambda(z+A\bar{z})} f(z)(dz + Ad\bar{z})\right| \leq$$

$$\leq \left| \begin{array}{l} z + A\bar{z} = Re^{i(\pi-\varphi)}, dz + Ad\bar{z} = -iRe^{i(\pi-\varphi)}d(\pi-\varphi), \\ \left|dz + Ad\bar{z}\right| = Rd(\pi-\varphi),\ 0 \leq \pi-\varphi \leq \frac{\pi}{2} \\ 0 \leq (\pi-\varphi) \leq \frac{\pi}{2},\ M(R) = \max_{R\to\infty}\left|f(z)\right| \end{array} \right|$$

$$\leq \int_{\gamma_R''} \left|e^{i\lambda(z+A\bar{z})}\right| \cdot \left|f(z)\right| \cdot \left|dz + Ad\bar{z}\right| \leq$$

$$\leq \int_0^{\frac{\pi}{2}} M(R) \cdot R \cdot e^{-\lambda R\frac{2}{\pi}(\pi-\varphi)} d(\pi-\varphi) =$$

$$= M(R) \cdot R \int_0^{\frac{\pi}{2}} e^{-\lambda R\frac{2}{\pi}(\pi-\varphi)} d(\pi-\varphi) =$$

$$= -M(R) \cdot R \cdot \frac{\pi}{\lambda R 2} e^{-\lambda R\frac{2}{\pi}(\pi-\varphi)}\Bigg|_0^{\frac{\pi}{2}} = M(R)\frac{\pi}{2\lambda}\left(1 - \frac{1}{e^{\lambda R}}\right) \xrightarrow[R\to\infty]{} 0$$

Proof of the theorem. As a result $\lim\limits_{R\to\infty} \int\limits_{\gamma_R} f(z)e^{i\lambda(z+A\bar{z})}(dz + Ad\bar{z}) = 0$

Equity is appropirate.

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