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Numerical determination of NN potential depth for a deuteron nucleus

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Abstract: Determining the potential depth of NN bound nuclei is important for determining the wave function of these nuclei. In particular, when determining the probability of the occurrence of nuclear reactions, the effective cross section and the reaction rate, it is necessary to know the wave function of the nuclei participating in this reaction. In this paper, we determined the depth of the NN potential of the deuteron nucleus (for Gaussian and exponential potentials) numerically using the wolfram mathematica software package.

Keywords: potential depth, NN potential, wave function.

Log in.

The Daitron core is a connected system consisting of one proton and one neutron. The Daitron nucleus is mainly caused by the ionization of the datery atom or the capture of slow neutrons from the proton tomo.

The main characteristics of the Daitron nucleus are:

Spin moment $S = 1$

Orbital moment $L = 0$

Magnetic moment , here's the Bor magnetoni$\mu_{deytron} = 0.857436\mu_B\mu_B$

Bonding Energy $E_b = -2.2245\ MeV$

For the Daitron core, the appearance of NN potential is important in the process of determining the potential depth of NN. In many publications, calculations have been made for the appearance of NN potential with a straight angle[1],[2]. For example, we can see$V_0 = 33.73\ MeV$ that Masatsugu Sei Suzuki's calculations amounted to the same as for the potential of a straight-angle shape.

We make calculations for situations where the appearance of the interaction potential of protons and neutrons in the Daitron core is as follows:

a) Potential exponentials

$$V_N(r) = -V_0 e^{-\frac{r}{R_N}} \qquad (1)$$

b) Gauss potensiali

$$V_N(r) = -V_0 e^{-\frac{r^2}{R_N{}^2}} \qquad (2)$$

Here is the potential depth of NN, and the potential parameter of NN.$V_0 R_N$

The main part.

In general, a system consisting of two choked nuclei is affected by the potential of the culon and the nucleus.

$$V(r) = V_c(r) + V_N(r) \qquad (3)$$

Same as here. As a core potential, we take the two potential mentioned above. We use the Schredenger equation to determine the depth of the potential intervening for the Daitron nucleus:$V_c(r) = \frac{z_1 z_2 e^2}{4\pi\varepsilon_0 r} V_0$

$$-\frac{h^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi \qquad (4)$$

The mass presented here is switched to the system of the inertia center. In our case and in accordance with the mass of protons and neutrons.$\mu = \frac{m_1 m_2}{m_1 + m_2} m_1 m_2$

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Shredenger tenglamasini spherical coordinatesada yechamiz chunki yadroviy potential markaziy simmetrik potential.

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right)-\frac{L(L+1)}{r^2}\psi\right)+V(r)\psi=E\psi \qquad (5)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right)-\frac{L(L+1)}{r^2}\psi+\frac{2\mu}{\hbar^2}\left(E-\frac{z_1z_2e^2}{4\pi\varepsilon_0 r}+V_N(r)\right)\psi=0 \qquad (6)$$

Let's do a replacement like in this equation. $\psi(\theta,\varphi,r)=\frac{\chi_L(r)}{r}Y_{L,m}(\theta,\varphi)$

$$\frac{d^2\chi_L(r)}{dr^2}-\frac{L(L+1)}{r^2}\chi_L(r)+\frac{2\mu}{\hbar^2}\left(E-\frac{z_1z_2e^2}{4\pi\varepsilon_0 r}+V_N(r)\right)\chi_L(r)=0 \qquad (7)$$

Let's clarify the boundary terms. If we mark it as (a very close distance to zero) $r=r_{min}r_{min}k_0=\sqrt{\frac{2\mu}{\hbar^2}(E+V_0)}$

$$\frac{d^2U_L(r)}{dr^2}+\left(k_0^2-\frac{L(L+1)}{r^2}\right)U_L(r)=0 \qquad (8)$$

If we mark it as in the resulting equation and perform a replacement $\rho=k_0rU_L=\rho^{\frac{1}{2}}\phi(\rho)$

$$\phi''+\frac{1}{\rho}\phi'+\left(1-\frac{\left(L+\frac{1}{2}\right)^2}{\rho^2}\right)\phi=0 \qquad (9)$$

This is tenglama spherical Bessel tenglama.

Otherwise the solution (10) $U_L(r)=\sqrt{\frac{\pi k_0 r}{2}}J_{L+\frac{1}{2}}(k_0r)$

So you can write the boundary terms for the radial solution to the above (7) equation:

$$\chi_L(r)|_{r=r_{min}}=U_L(r_{min}), \qquad \chi_L{}'(r)|_{r=r_{min}}=k_0U_L{}'(r_{min}) \qquad (11)$$

There is such a distance that at greater distances than this point, the nuclear potential is zero (Fig. 6). $r=R_0$

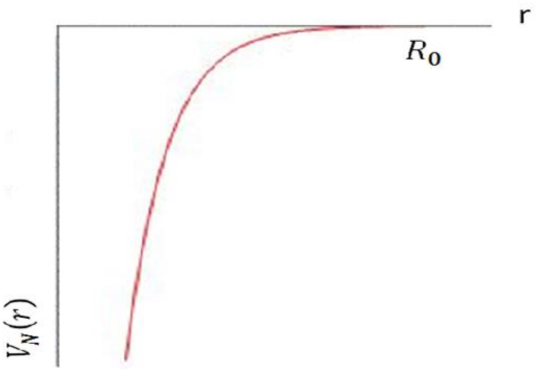

Figure 1. An approximate view of the distance dependence of the nucleus potential of two nuclei.

That is, beyond this point, the Culon field plays an important role. Let's find out what (7) the solution to the equation will look like outside of this point.

In the area we are seeing (7) the view of the equation is as follows:

$$\frac{d^2\xi_L(r)}{dr^2}-\frac{L(L+1)}{r^2}\xi_L(r)+\frac{2\mu}{\hbar^2}\left(E-\frac{z_1z_2e^2}{4\pi\varepsilon_0 r}\right)\xi_L(r)=0 \qquad (12)$$

$k=\sqrt{\frac{2\mu E}{\hbar^2}}$ and we're going to mark it.

$$\frac{d^2\xi_L(r)}{dr^2}-\frac{L(L+1)}{r^2}\xi_L(r)+k^2\xi_L(r)-\frac{2\mu z_1z_2e^2}{\hbar^2 4\pi\varepsilon_0}\frac{\xi_L(r)}{r}=0 \qquad (13)$$

Let's divide this equation into: $4k^2$

$$\frac{d^2\xi_L(r)}{d(2kr)^2}-\frac{L(L+1)}{(2kr)^2}\xi_L(r)+\frac{1}{4}\xi_L(r)-\frac{\mu z_1z_2e^2}{k\hbar^2 4\pi\varepsilon_0}\frac{\xi_L(r)}{2kr}=0 \qquad (14)$$

Considering it's $v=\frac{\hbar k}{\mu}$

$$\frac{d^2\xi_L(r)}{d(2kr)^2}-\frac{L(L+1)}{(2kr)^2}\xi_L(r)+\frac{1}{4}\xi_L(r)-\frac{z_1z_2e^2}{4\pi\varepsilon_0\hbar v}\frac{\xi_L(r)}{2kr}=0 \qquad (15)$$

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If we enter the Zommerfeld parameter, the equation will look like this:$\eta_0 = \frac{z_1 z_2 e^2}{4\pi\varepsilon_0 \hbar v}$

$$\frac{d^2\xi_L(r)}{d(2kr)^2} - \frac{L(L+1)}{(2kr)^2}\xi_L(r) + \frac{1}{4}\xi_L(r) - \eta_0\frac{\xi_L(r)}{2kr} = 0 \quad (16)$$

$\rho = 2kr$ we enter a markup:

$$\frac{d^2\xi_L(r)}{d(\rho)^2} - \frac{L(L+1)}{(\rho)^2}\xi_L(r) + \frac{1}{4}\xi_L(r) - \eta_0\frac{\xi_L(r)}{\rho} = 0 \quad (17)$$

Let's replace the view by entering a markup of this equation. As a result, the equation will be as follows.$m = L + \frac{1}{2}$

$$\left(\frac{d^2}{d\rho^2} - \frac{1}{4} + \frac{\eta_0}{\rho} + \frac{\frac{1}{4} - m^2}{\rho^2}\right)\xi_L(r) = 0 \quad (18)$$

This equation is the Witteker equation itself. Therefore, the solution

$$\xi_L(r) = C_1 W_{-\eta_0, L+\frac{1}{2}}(2kr) + C_2 W_{\eta_0, L+\frac{1}{2}}(-2kr) \quad (19)$$

in the form. Here and the larynx are called the Witteker function. This function (1) is the only asymptotics of the equation at great distances, i.e. $W_{-\eta_0, L+\frac{1}{2}}(2kr) W_{\eta_0, L+\frac{1}{2}}(-2kr)$

$$\chi_L{}^{as}(r) = \xi_L(r) = C_1 W_{-\eta_0, L+\frac{1}{2}}(2kr) + C_2 W_{\eta_0, L+\frac{1}{2}}(-2kr) \quad (20)$$

Asymptosis of the Witteker function over large distances has the following view:

$$W_{-\eta_0, L+\frac{1}{2}}(2kr) \to \frac{e^{-kr}}{(2kr)^{\eta_0}} \quad (21)$$

$$W_{\eta_0, L+\frac{1}{2}}(-2kr) \to e^{kr}(2kr)^{\eta_0} \quad (22)$$

The graph of these functions looks like this (Figure 7).

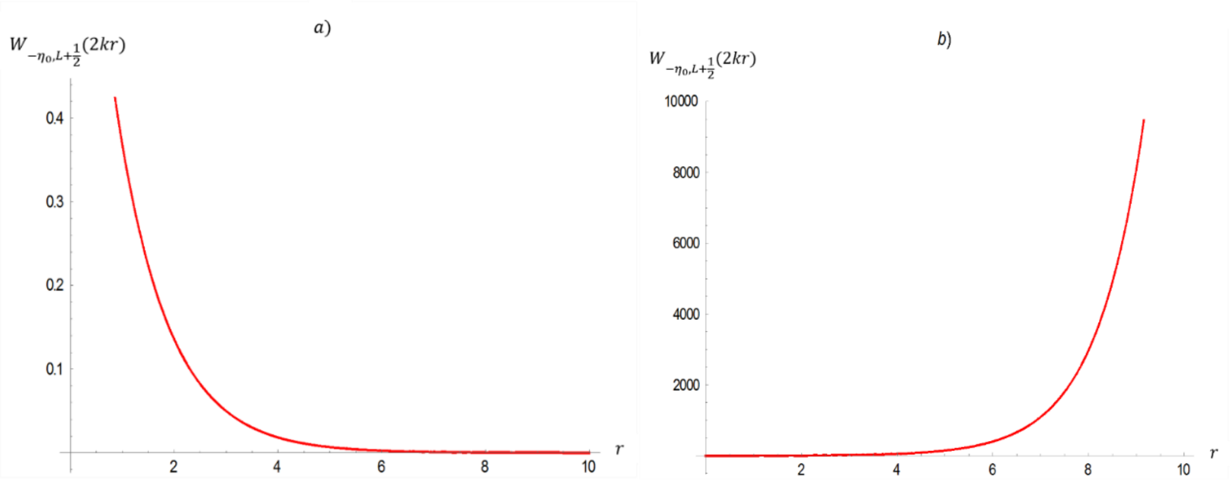

Figure 2. Remote link graph of Witteker function asymptotics.

As you can see from the 2b chart, at great distances the function seeks unlimited. Based on the fundamental laws of quantum mechanics, wave function must be limited. $W_{\eta_0, L+\frac{1}{2}}(-2kr)$

We take two very closely related such and points that (at these points and the condition is met, the main role is performed by the Culon potential) in which (7) the solution to the equation$r_1 r_2 r_1 > R_0 r_2 > R_0$

$$\chi_L(r_1) = C_1 W_{-\eta_0, L+\frac{1}{2}}(2kr_1) + C_2 W_{\eta_0, L+\frac{1}{2}}(-2kr_1) \quad (23a)$$

$$\chi_L(r_2) = C_1 W_{-\eta_0, L+\frac{1}{2}}(2kr_2) + C_2 W_{\eta_0, L+\frac{1}{2}}(-2kr_2) \quad (23b)$$

will be in the form. Of these, we determine the ratio.$d = \frac{C_2}{C_1}$

$$\frac{\chi_L(r_1)}{\chi_L(r_2)} = \frac{W_{-\eta_0, L+\frac{1}{2}}(2kr_1) + dW_{\eta_0, L+\frac{1}{2}}(-2kr_1)}{W_{-\eta_0, L+\frac{1}{2}}(2kr_2) + dW_{\eta_0, L+\frac{1}{2}}(-2kr_2)} \quad (24)$$

$$d = \frac{\chi_L(r_1) W_{-\eta_0, L+\frac{1}{2}}(2kr_2) - \chi_L(r_2) W_{-\eta_0, L+\frac{1}{2}}(2kr_1)}{\chi_L(r_2) W_{\eta_0, L+\frac{1}{2}}(-2kr_1) - \chi_L(r_1) W_{\eta_0, L+\frac{1}{2}}(-2kr_2)} \quad (25)$$

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Because for the wave function to be limited, this size must be very close to zero. Now, if we change the value of the potential depth from a starting value and (7) determine the solution to the equation and in a number of ways at points, (25) the value of the expression will also change and be very close to zero at any point. The value of this point gives us the potential depth we are looking for. Calculations were made using the Wolfram mathematica software package, and the following results were obtained: $V_0 \chi_L(r) r_1 r_2 d d V_0$

Potensiallar / Parametrlar	Exponential potential	Gauss potensiali
R_N	$0.683\ fm$	$1.65\ fm$
V_0	$184.08\ MeV$	$60.5713\ MeV$

The abstract.

The NN potential depth for the Daitron core was calculated for exponential and Gauss potential. The calculation was carried out in a final way using the wolfram mathematica application package. Since the Daitron nucleus is a strangled nucleus, the wave function must be limited at great distances. Therefore, in the process of determining potential depth, we used an asymptotic solution to the Shredenger equation.

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