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Bilateral Compression and Bending of a Plate with All Edges Elastically Fixed (Approximate Solution)

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Abstract. *In this paper, we consider the solution to the problem of a plate elastically fixed at all edges with thin-walled open-profile rods and subjected to simultaneous bilateral compression and transverse loads of intensity q.*
Keywords: compression, bending, plate, elastically hinged edges, torsion, stiffness.

Problem statement and literature review. Accounting for constrained torsion in thin-walled open-section rods enables a more thorough understanding of the potential to enhance the torsional rigidity of these rods. The impact of constrained torsion enhances the effectiveness of plate edge fixing when thin-walled open-section rods serve as reinforcing elements.

The method for establishing boundary conditions along the contact line between a plate and its reinforcing ribs is detailed in references [2, 3, 4, 6]. The loads transmitted from the plate to the ribs (or rods) are considered equal in magnitude but opposite in direction to the forces in the corresponding sections of the plate. These force conditions are further supplemented by kinematic conditions to ensure that displacements at the points of contact between the reinforcing rods and the plate are equal.

References [1, 5] present a method for developing refined boundary conditions at the plate-rod interface, which takes into account the warping constraints of the end sections of the ribs. The degree of warping constraint is represented by parameter d_k.

Solution method. In many situations, it is appropriate to find a simpler, approximate solution to a given problem. To achieve this, we will utilize the well-known Bubnov-Galerkin approximate method. This method, as opposed to finding an exact solution, enables us to determine the critical load intensity N^* explicitly, without the need to solve complex transcendental equations.

Let us consider the solution to the problem of a plate elastically fixed at all edges and subjected to the simultaneous action of bilateral compression and transverse loads of intensity q.

We assume that all edges are fastened to thin-walled rods, ignoring the deflections of the latter ($t_u = \infty$). Assuming the rods at parallel edges to be identical, we denote the elastic restraint coefficients by t_{k_1} (at edges $y = \pm\frac{b}{2}$) and t_{k_2} (at edges $x = \pm\frac{a}{2}$), respectively.

The differential equation of the problem has the following form:

$$D\nabla^2\nabla^2 w = N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} - q(x,y) = 0. \tag{1}$$

The boundary conditions of elastic fixing of all edges under consideration do not allow integration of equation (1) in closed form. Therefore, we apply the Bubnov-Galerkin method:

$$\int_{-a/2}^{+a/2}\int_{-b/2}^{+b/2} L(w) w dx dy = 0, \tag{2}$$

where $L(w)$ - is the left-hand side of equation (1).

Below, to make the problem more concrete, we assume that $q = const$ and, using symmetry, place the origin at the center of the plate (Fig. 1).

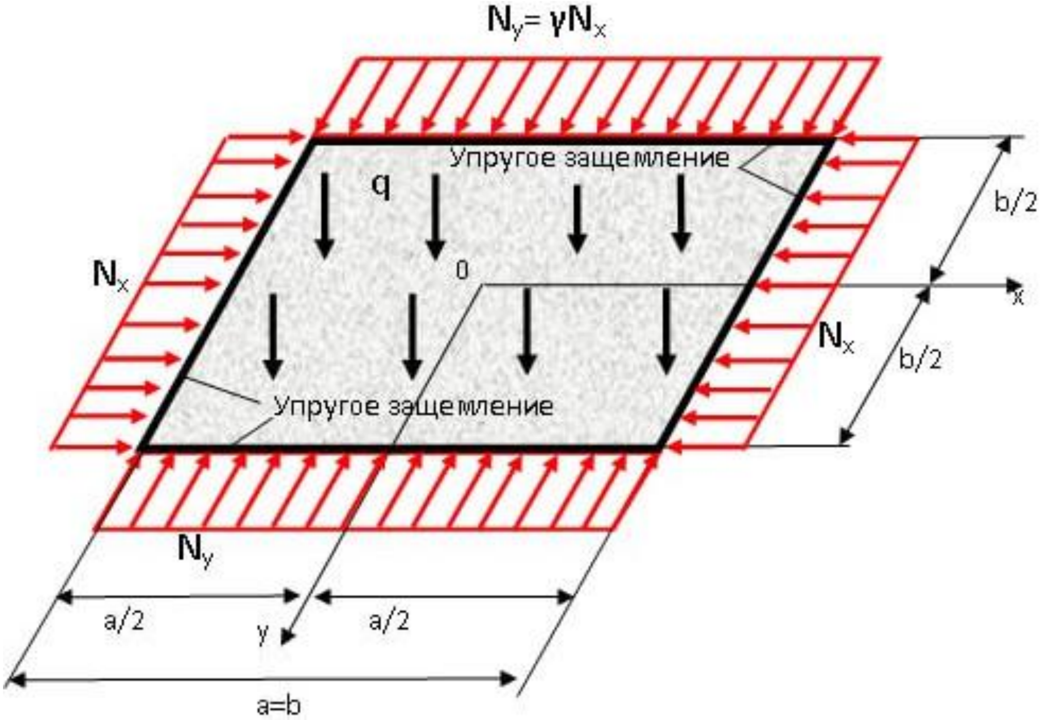

Fig. 1. Bilateral compression and bending of plates

Boundary conditions under elastic fixing at all edges are:

$$f_n^I = \frac{\mu\lambda_n^2 f_n - f_n^{II}}{t_k b \lambda_n^2}, \tag{3}$$

where
$$t_k = \frac{C_k}{d_k}\left(1 + \frac{\lambda_n^2}{k^2}\right), \quad C_k = \frac{GJ_k}{Db}, \quad k^2 = \frac{GJ_k}{EJ_\omega}, \tag{4}$$

$$d_k = 1 \text{ for } \frac{d^2\theta}{dx^2}\Big|_{x=0,a} = 0, \tag{5}$$

$$d_k = 1 - \frac{2ka}{n^2\pi^2 - k^2a^2}\left\{\frac{[1-(-1)^n]\,\mathrm{sh}\,ka}{chka-1} + \frac{[1+(-1)^n]ka(1-chka)}{2(chka-1)-kashka}\right\} \text{ for } \frac{d\theta}{dx}\Big|_{x=0,a} = 0\,. \tag{5a}$$

The approximating function satisfying all boundary conditions has the following form:

$$w(x,y) = f_1\left(1 + \frac{\pi b^2}{2a^2}t_{k_1}cos\frac{\pi y}{b}\right)\left(1 + \frac{\pi a^2}{2b^2}t_{k_2}cos\frac{\pi x}{a}\right)cos\frac{\pi x}{a}cos\frac{\pi y}{b}\quad . \tag{6}$$

As a result, from (2), we obtain

$$w(x,y) = \frac{48(1-\mu^2)qb^4}{\pi^5 E\delta^3}\left[\frac{A_6A_7cos\frac{\pi x}{a}cos\frac{\pi y}{b}}{A_1 + A_2 + A_3 - 4\frac{N_x}{N_y}(A_4 + \gamma A_5)}\right], \tag{7}$$

Where,

$$N_y = \gamma N_x, \qquad N_э = 4\pi^2 D/b^2 \tag{8}$$

$$A_1 = \frac{\pi^2 b^3}{4a^3}\left(1 + \frac{20a^2}{3b^2}t_{k_2} + \frac{\pi^2a^4}{b^4}t_{k_2}^2\right)\left(1 + \frac{8b^2}{3a^2}t_{k_1} + \frac{3\pi^2b^4}{16a^4}t_{k_1}^2\right)$$
$$A_2 = \frac{\pi^2 b}{2a}\left(1 + \frac{8a^2}{3b^2}t_{k_2} + \frac{\pi^2a^4}{4b^4}t_{k_2}^2\right)\left(1 + \frac{8b^2}{3a^2}t_{k_1} + \frac{\pi^2b^4}{8a^4}t_{k_1}^2\right)$$
$$A_3 = \frac{\pi^2 a}{4b}\left(1 + \frac{8a^2}{3b^2}t_{k_2} + \frac{3\pi^2a^4}{16b^4}t_{k_2}^2\right)\left(1 + \frac{20b^2}{3a^2}t_{k_1} + \frac{\pi^2b^4}{a^4}t_{k_1}^2\right) \tag{9}$$
$$A_4 = \frac{\pi^2 b}{4a}\left(1 + \frac{8a^2}{3b^2}t_{k_2} + \frac{\pi^2a^4}{4b^4}t_{k_2}^2\right)\left(1 + \frac{8b^2}{3a^2}t_{k_1} + \frac{3\pi^2b^4}{16a^4}t_{k_1}^2\right)$$
$$A_5 = \frac{\pi^2 a}{4b}\left(1 + \frac{8a^2}{3b^2}t_{k_2} + \frac{3\pi^2a^4}{16b^4}t_{k_2}^2\right)\left(1 + \frac{8b^2}{3a^2}t_{k_1} + \frac{\pi^2b^4}{4a^4}t_{k_1}^2\right)$$
$$A_6 = \frac{a}{b}\left[1 + t_{k_1}\frac{\pi^2b^2}{a^2}\left(1 + \frac{1}{2\pi}\right) + t_{k_1}^2\frac{\pi^3b^4}{2a^4}\right],$$
$$A_7 = 1 + t_{k_2}\frac{\pi^2a^2}{b^2}\left(1 + \frac{1}{2\pi}\right) + t_{k_2}^2\frac{\pi^3a^4}{2b^4}$$

Solution (7) is general; from it, as special cases, solutions to a number of simpler problems follow. For example, for $N_x = N_y = 0$, we obtain the problem of transverse bending of a plate with elastically fixed edges; in this case, Eq. (7) takes the following form:

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June 2026

$$w(x,y) = \frac{48(1-\mu^2)qb^4}{\pi^5 E\delta^3}\left[\frac{A_9 A_{10} cos\frac{\pi x}{a} cos\frac{\pi y}{b}}{A_4+A_5+A_6}\right]. \tag{10}$$

If we set $q = 0$ in Eq. (7), we arrive at the problem of stability of an elastically fixed plate under bilateral compression. Equating the denominator in square brackets to zero in Eq. (7), we arrive at the previously obtained expression:

$$N_x^* = \frac{\pi^2 D}{b^2}\frac{A_4+A_5+A_5}{A_7+\gamma A_8}, \qquad \gamma = \frac{N_y^*}{N_x^*}.$$

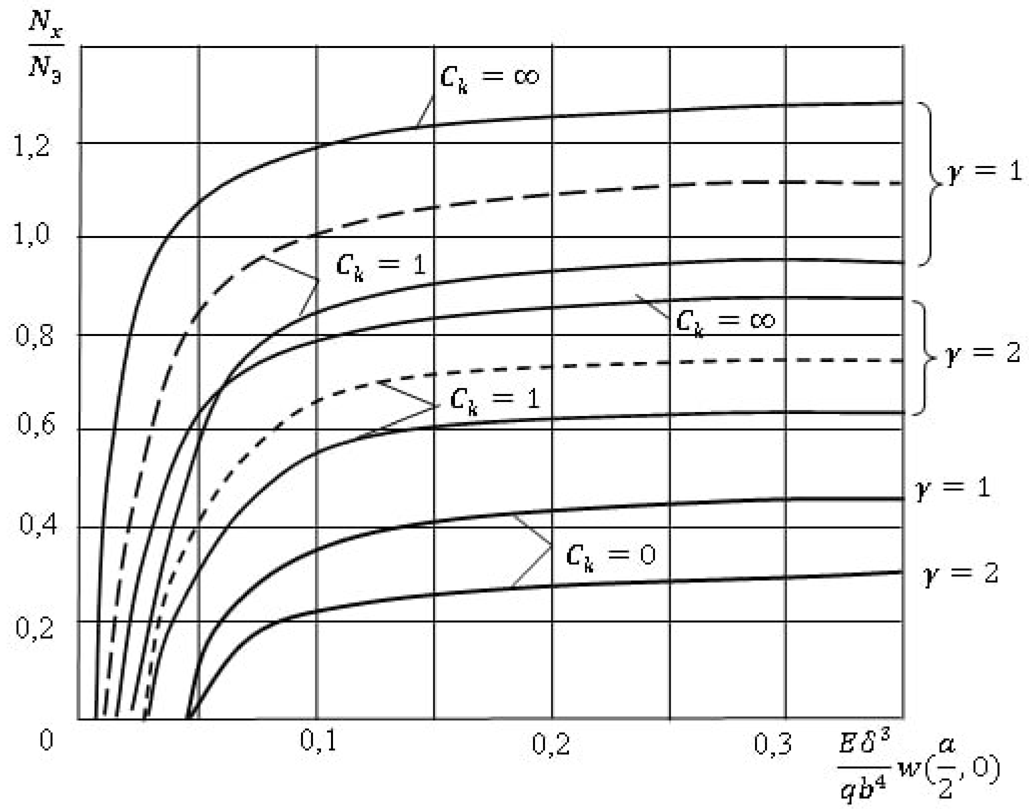

Fig. 2. Graph of the plate center deflection versus N_x for various values of γ.

In addition to the above, other special cases follow from the general solution under various assumptions on values t_{k_1}, t_{k_2}. For example, let $t_{k_2} = 0$, i.e., the plate is hinged at the edges $x = \frac{a}{2}$. Then, from (7), we obtain:

$$w(x,y) = \frac{48(1-\mu^2)qb^4}{\pi^5 E\delta^3}\left[\frac{A_7 cos\frac{\pi x}{a} cos\frac{\pi y}{b}}{A_8+A_9+A_{10}-4\frac{N_x}{N_э}(A_{11}+\gamma A_{12})}\right], \tag{11}$$

Where,

$$\begin{aligned}
A_8 &= \frac{\pi^2 b^3}{4a^3}\left(1+\frac{8b^2}{3a^2}t_{k_1}+\frac{3\pi^2 b^4}{8a^4}t_{k_1}^2\right),\\
A_9 &= \frac{\pi^2 b}{2a}\left(1+\frac{8b^2}{3a^2}t_{k_1}+\frac{\pi^2 b^4}{8a^4}t_{k_1}^2\right),\\
A_{10} &= \frac{\pi^2 a}{4b}\left(1+\frac{20b^2}{3a^2}t_{k_1}+\frac{\pi^2 b^4}{a^4}t_{k_1}^2\right),\\
A_{11} &= \frac{\pi^2 b}{4a}\left(1+\frac{8b^2}{3a^2}t_{k_1}+\frac{3\pi^2 b^4}{a^4}t_{k_1}^2\right),\\
A_{12} &= \frac{\pi^2 a}{4b}\left(1+\frac{8b^2}{3a^2}t_{k_1}+\frac{\pi^2 b^4}{4a^4}t_{k_1}^2\right).
\end{aligned} \tag{12}$$

Let us illustrate some results derived from the general formula (7). We consider identical thin-walled rods along all edges, i.e., $t_{k_1} = t_{k_2} = t_k$.

In this scenario, we plotted a graph of the deflection at the center of the plate against N_x for various values of γ, see Figure 2. The solid lines represent the case of free warping at the ends of the reinforcing rods, while the dashed lines indicate complete constraint. The following values were assumed for the analysis: $a = b,\ n = 1,\ ka = 8,\ \mu = 0{,}3$.

ISSN NO: 2770-4491
June 2026

From the graph, it is evident that a relatively weak fixation of the plate edges (for example, $C_k = 1$) significantly increases the plate's rigidity compared to a scenario where the edges are simply hinged.

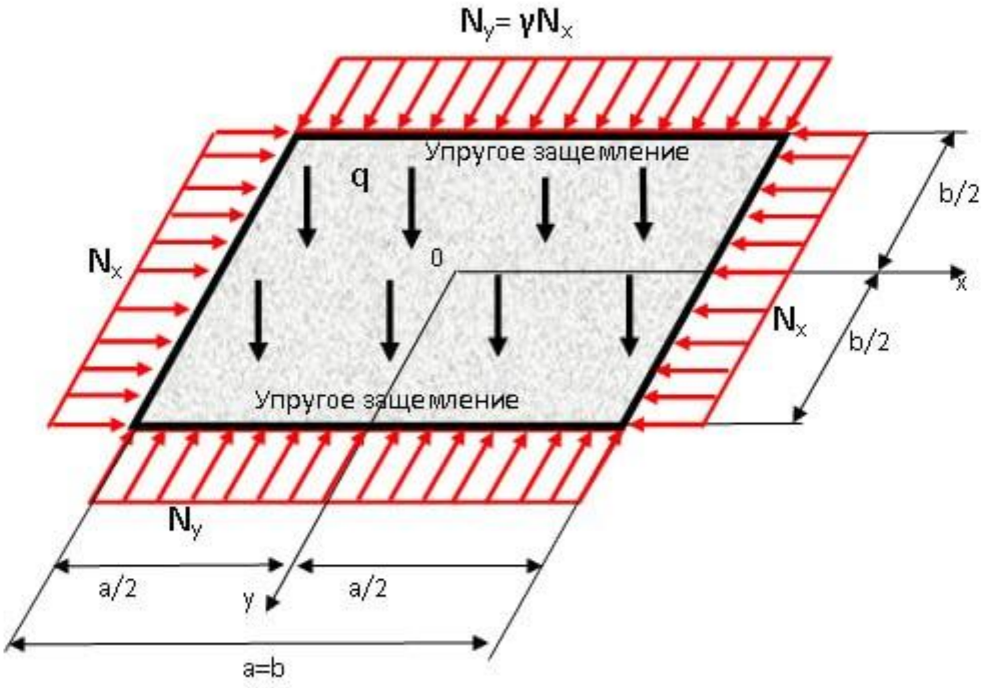

Fig. 3. Bilateral compression and bending of plates

Figure 4 shows a similar graph, constructed using formula (11), for a plate with two edges (for $x = \pm\frac{a}{2}$) supported by hinges. Solid lines refer to the free deplaning of the ends of the reinforcing rods, dashed lines refer to their complete constriction. When constructing, it is assumed: a=b; n=1; ka=8; μ=0.3.

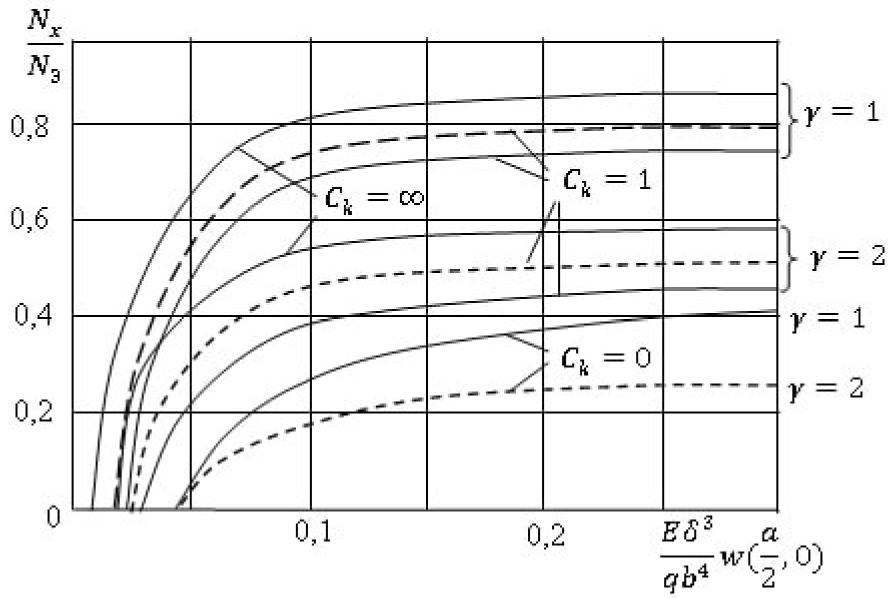

Fig. 4. Graph of the plate center deflection versus N_x for various values of γ.

ISSN NO: 2770-4491
June 2026

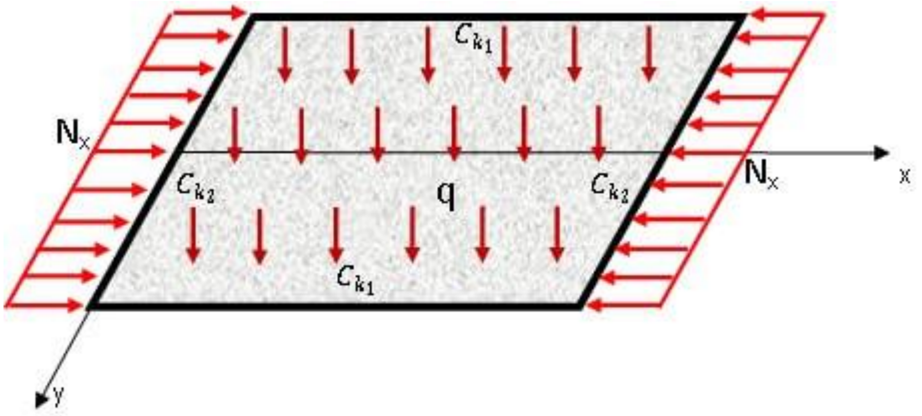

Fig. 5 Unilateral compression of the plates.

Finally, Figure 5 shows a graph for the special case, $\gamma = 0$ (unilateral compression). Here, the longitudinal forces are applied to the edges, for which the fixing coefficient is set (see diagram in Figure 5). The fixation stiffness of the other two edges ($y = \pm\frac{b}{2}$) varies from zero to infinity. Solid lines refer to the free deplaning of the ends of the reinforcing rods, dashed lines refer to their complete constriction. When constructing, it is assumed: a=b; n=1; ka=8; μ=0.3.

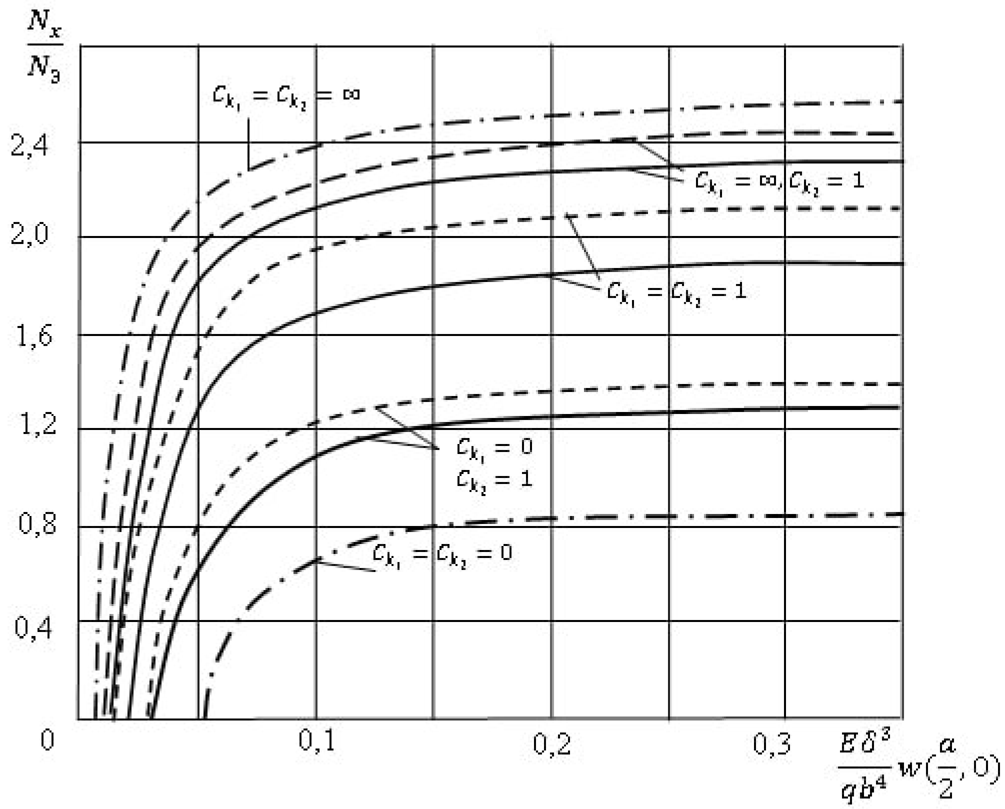

Fig. 6. For comparison, curves (dashed line) are shown for the two extreme cases of the sides of the plate resting.

For comparison, Fig. 6 shows curves (dashed and dotted lines) for two extreme cases of plate side support: the upper curve refers to stiff fixing of all edges and the lower curve refers to their hinged support.

ISSN NO: 2770-4491
June 2026

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