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Determination of Gas Pressure Distribution in a Pipeline Network Using the Broyden Method

Mamajonov Muxammadaziz Abdirashid o`g`li
4-course student of
Andijan machine-building institute
Uzbekistan, Andijan
Mamajonov1700@gmail.com
+998 90 040 17 00
Solijonov Shaxboz Mashrabjon o`g`li
4-course student of
Andijan machine-building institute
Uzbekistan, Andijan
Hamraqulov Hotambek Hakimjon o`g`li
Assistant,
Andijan machine-building institute,
Uzbekistan, Andijan
slove1994s@gmail.com
+998 93 427 71 24

Abstract. A potential problem in natural gas pipeline networks is bottlenecks occurring in the flow system due to unexpected high pressure at the pipeline network junctions resulting in inaccurate quantity and quality (pressure) at the end user outlets. The gas operator should be able to measure the pressure distribution in its network so the consumers can expect adequate gas quality and quantity obtained at their outlets. In this paper, a new approach to determine the gas pressure distribution in a pipeline network is proposed. A practical and userfriendly software application was developed. The network was modeled as a collection of node pressures and edge flows. The steady state gas flow equations Panhandle A, Panhandle B and Weymouth to represent flow in pipes of different sizes and a valve and regulator equation were considered. The obtained system consists of a set of nonlinear equations of node pressures and edge flowrates. Application in a network in the field involving a large number of outlets will result in a large system of nonlinear equations to be solved. In this study, the Broyden method was used for solving the system of equations. It showed satisfactory performance when implemented with field data.

Keywords: Broyden method; gas pipeline network; pressure distribution; steady state gas flow.

Introduction. Natural gas is widely used as a source of energy for industrial needs and public household consumption. Gas operators have the responsibility to provide gas to their consumers at a certain rate and pressure at their request. The gas operator should be able to preserve the gas pressure distribution and flowrate at each outlet or the consumer's entry point. There are two main problems in natural gas distribution networks: optimization of pipeline diameter and determination of pressure distribution. Some research on pipeline diameter optimization can be found in [1-3], while some research on the determination of the pressure distribution in transmission networks can be found in [4], and more recently in [5].

The author in [5] expresses concern about the scarceness of methods for flow computation for gas networks in the presence of multiple pressure levels. This feature is important in the analysis of real gas systems, where most of the observed networks cannot be decomposed into pressure-homogeneous portions, so they will be solved independently. In the same paper, a steady-state flow formulation with multiple pressure levels is proposed and implemented into a gas distribution network containing 67 nodes and 88 edges. It also takes into account corrections for elevation changes in the pipes.

The present study focused on determining gas pressure distribution in pipeline networks that have multiple sources with multiple pressures. The network was considered as connected pipelines with steady-state gas flow from one or more supply points to one or more delivery points. Also, the flow in valves and regulators

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was represented by an equation. Hence we have a system of nonlinear equations with several variables that constitute the system model. The Newton and quasi-Newton methods, which are widely used to iteratively solve systems of nonlinear equations, have an advantage in their speed of convergence once they are given a sufficiently accurate initial guess of the root. Nowadays, the most commonly used approach is to run an optimization method first to find the desired initial guess and then feed it to the Newton method. This hybrid approach has been proven to be more satisfactory than using the root finding method solely. Luo, *et al.* [6] proposed a hybrid approach using a chaos optimization algorithm and the quasi-Newton method, while Burden and Faires [7] used a combination of the steepest descent method and the Newton method for solving the nonlinear equation system. The latest approach is given by Sidarto & Kania in [8]. Because solving nonlinear equation systems is related to the pressure distribution in gas pipeline distribution networks, Sidarto, [9,10] have proposed a genetic algorithm optimization method combined with the Newton method. It uses the genetic algorithm to obtain a good initial guess of the root that is used by the Newton-type method to obtain the solution of the nonlinear equation system. However, for large systems it is observed that the convergence of the optimization process before applying the Newton-type method is rather slow. Detailed information on converting the root finding problem to an optimization process can be found in [8].

Using the Newton method for solving a system of $\boldsymbol{n}$ nonlinear equations with $\boldsymbol{n}$ variables, not only the function definitions must be provided but also the $\boldsymbol{n}^2$ partial derivatives of the functions at each iteration. For large systems this is certainly a disadvantage of the method. The Broyden method avoids the calculation of those partial derivatives. In the present research, Broyden's method was used so matrix inversion does not need to be computed at each step [11]. The details will be explained in Section 4. Nowadays, this method is used in many applications. According to [12], the industrial practice of branched gas transmission network (GTN) analysis and operation requires high-accuracy computational fluid dynamics (CFD) simulators. The numerical solution of the obtained system of equations was performed by the modified Broyden method, which has been proven to be one of the best performing extensions of the classical secant method for numerical solution of non-linear algebraic equations. In [13], the development of strongly nonlinear problems in helicopter aeroelasticity is considered. For strongly nonlinear problems, numerical solutions obtained in an iterative process can diverge due to numerical instability. Therefore, choosing the method is critical. In the same research, a comparative study was conducted using the modified Newton, rank-1 Broyden and rank-2 BFGS (Broyden-Fletcher-Goldfarb-Shanno) update methods. One of the results showed that Broyden's update method gives a reduction of the number of iterations relative to the Newton method and it gives a higher rate of convergence. The convergence of the Broyden method has been extensively studied in [14] and [15].

The obtained result of this new approach was compared to the result from TGNet, a commercial software application commonly used in natural gas pipeline network simulation. The software application is suitable for static pipeline network simulation [16]. According to [17], it gives better performance compared to other well-known software applications in single-phase gas flow simulation, such as OLGA and SPS. The new approach was developed as a new software application, called DistNet by OPPINET, in which the initial values are generated randomly and the calculation of solutions for the system of equations is conducted using the Broyden method. It will be shown that its performance is as good as that of TGNet, which validates the results of DistNet. This software application has some features that are not found in TGNet, such as providing a choice between meta-heuristic methods combined with the Newton or quasi-Newton method so its running time can be managed to give better performance.

Methodology. A complex gas pipeline network can be modeled as a directed and connected graph (V,E) , in which $V = \{ v_i \mid i=1,2\ldots ,N\}$ is the set of vertices/nodes consisting of inlets, outlets, and junctions V =V_I□V_O□□V_J . E is the set of directed edge connecting nodes representing the flows' directions, so it consists of pipes, valves, and regulators. Each flowrate that connects node $\boldsymbol{i}$ and node $\boldsymbol{j}$ is governed by the steady empirical gas flow equation and also by the valve and regulator equation. The constraints used in the determination of the flowrate and pressure distribution are the balancing equations, which are based on Kirchhoff's law. In solving the obtained system of equations, we use the Broyden numerical method, which is a quasi-Newton method for finding the roots of a system of N nonlinear equations for N variables. The

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Newton method for solving the system needs the computation of the Jacobian matrices at every iteration, which is a difficult and expensive operation for the system when N is quite large. The Broyden method computes the Jacobian matrix only once, at the first iteration, and does a rank-one update at the rest of other iterations.

Gas Flowrate Equations on Pipes and Valves/Regulators

A pipeline system consists of nodes and node-connecting elements (NCE). Nodes represent points where one or more NCEs terminate and where a gas flow enters or leaves the system. Nodes are also the reference points for the pressures of the system. Several types of NCE commonly used in networks are pipelines, compressors, valves, and regulators. In this study, all these types were considered, with the exception of compressors. In balancing the flowrates using a mathematical model, a steady-state model from the continuity equation at each node in the system was used [18]. The Weymouth, Panhandle A and Panhandle B gas flow equations were used to represent flow in different sizes of pipes. The most common pipeline flow equation is the Weymouth equation, which is generally preferred for transmission line diameters smaller than 15 inch. The other equations are usually better for larger-sized transmission lines. These equations were developed to simulate compressible gas flows in long pipelines [19]. A pipe that connects node i and node j has length L_{ij} (mile), with inside diameter ID_{ij} (inch), average flowing temperature T_{ij} (°F), specific gravity G_{ij} and pipe efficiency E_{ij} The pipeline system in this research was assumed to be in steady-state condition. The flow from node $\boldsymbol{i}$ to $\boldsymbol{j}$ is expressed as a flow with a positive signed value. The gas flowrate is expressed in units of MMSCFD (million standard cubic feet of gas per day) and the gas pressure is expressed in units of psia (pounds per square inch absolute).
For horizontal flow, the general flowrate equation in a pipeline is written as follows [19]:

$$Q_{ij} = a_1\, E_{ij} \left(\frac{T_b}{P_b}\right)^{a_2} \left(\frac{P_i^2 - P_j^2}{T_{ij}\, z\, L_{ij}}\right)^{a_3} \left(\frac{1}{G_{ij}}\right)^{a_4} ID_{ij}^{a_5}$$

where Q_{ij} is the volumetric gas flowrate in a pipe that connects nodes $\boldsymbol{i}$ and $\boldsymbol{j}$. P_i and P_j are the pressures at nodes $\boldsymbol{i}$ and $\boldsymbol{j}$ respectively. T_b and P_b are the base temperature and pressure respectively**.** $\mathbf{Z}$ is the gas deviation factor at average flowing temperature and average pressure.
Broyden Numerical Methods. The Broyden method is for numerically solving a nonlinear system of equations and is derived from the Newton method [7]. Consider a system of nonlinear equations $F(x)=0$ and a given initial approximate solution P_o. The method generates a sequence$\{P_k\}$ that will converge to $\mathbf{p}$ such thatF(p)=0. For the Newton method and $k\geq0$ first compute $F(p_k)$ and the Jacobian matrix $J(p_k)$ then find Δp that satisfies

$$J(p_k)\ \Delta=-F(p_k)$$

So we have the following iteration formula:

$$p_{k+1}=p_k+\Delta p$$

The Broyden method replaces the computationally expensive Jacobian matrix $J(p_k)$ with a simple choice matrix A_k Initially the method sets $A_o=J(P_o)$For the computation of matrices A_k $k>0$ as the next replacement of the Jacobian matrices the following update is used:

$$A_k = A_{k-1} + \frac{1}{S_k{}^{T}\, S_k}\left(Y_k - A_{k-1}S_k\right) S^{T}{}_k$$

where $S_k=p_k - p_{k-1}$ and $Y_k = F(p_k) - F(p_{k-1})$
So we find Δp satisfies :
$A_k\ \Delta p=-F(p_k)$,
and the method from Eq (4) gives:
$p_{k+1}=p_k-A_k^{-1}F(p_k)$.
Furthermore, the Sherman-Morison matrix inversion formula (see for example [7]) is used to compute A_k^{-1} from A_{k-1}^{-1} by the following formula:

$$A_k^{-1} = A_{k-1}^{-1} + \frac{1}{\vec{S}_k{}^{T} A_{k-1}^{-1}\vec{Y}_k}\left(\vec{S}_k - A_{k-1}^{-1}\vec{Y}_k\right) \vec{S}_k{}^{T} A_{k-1}^{-1}\ ,$$

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eliminating the need of a matrix inversion at each iteration.

Conclusion. The method developed in the novel software application DistNet was shown to have the ability to simulate a natural gas distribution network with pipes, valves, and regulators. By using the Broyden method, the pressure distribution and flow direction can be obtained within a short period of simulation time. Note that the direction of the flow does not have to be defined in the input data because the model can determine the direction naturally as gas flow will go from higher pressure to lower pressure. If we have very high demand at one outlet, for example outlet-13 in the observed network, the pressures will be high along all nodes leading to that outlet. The results of the DistNet software application are very close to those of TGNet. The average differences in pressure distribution for Panhandle A, Panhandle B and Weymouth Correlations are 0.082% 0.6818%, and 1.8655% respectively.
A simplified network without valves and regulators can be more efficient than the original network. In order to fulfil the same flowrate demands at all outlets, the source pressure can be reduced up to 30% of the original value. On the other hand, the simplified system with original source pressure can supply up to 3.8 times the original demand. Comparing the simulation results of the original and the simplified network, the existing valves and regulators installed in the original network seem to create inefficiency. However, conditions in the field sometimes create unexpected constraints, so the valves and regulators are still needed to control the pressures and flowrates.

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