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Requirements for Approximator Functions

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Abstract: The main methods of approximation of magnetization curves are considered in the article. Requirements for approximating functions are given. Basic and approximated magnetization curves for various types of approximating functions were constructed and their deviation from experimental magnetization curves was determined. Conclusions on the choice of approximating functions and approximation methods are given.

Key words: magnetization curve, exponential approximation, spline approximation, rational function, degree polynomial, asymptotic approximation.

Ferromagnetic electrotechnical materials are widely used in various fields. When using these electrotechnical materials, it is necessary to know their basic technical parameters. It is necessary to calculate the power dissipation occurring in the steel core of electromagnetic devices and various ferromagnetic elements and to select the normal operating mode of this object or to determine the cumulative currents and magnetic field distribution in the ferromagnetic material. Solving these problems cannot be achieved without taking into account the nonlinearity of any ferromagnetic material characteristic.

Functions of various forms are used to analytically represent the magnetization curve in the zone where the magnetic field can change. If the magnetic field strength-*H* and magnetic field induction-*B* do not change their signs, such a function can be odd or even. Such a function can only be odd if *H* and *B* change their signs. The approximation coefficients of these functions are determined based on selected points or least squares methods.

Several methods have been developed to approximate the magnetization curves of various ferromagnetic materials. The following requirements are imposed on these approximating functions:

gives a result close to the experimental magnetization curve;

taking into account that the function can be used in the process of differentiation, its derivative also gives a close result to the magnetization curve;

to be relatively simple in appearance;

be an odd function;

include universal and magnetization curve parameters;

that the function does not have an inflection point on each semi-axis;

the function should consist of as few constants as possible.

Piecewise approximation. This method is used to calculate magnetic conductors with low non-linearity of the material from which the device is made. In this case, the approximating curve is replaced by broken lines consisting of one or more straight lines. The number of approximated sections depends on the required calculation accuracy and the range of magnetic field variation. For example, if the material is close to the saturation medium, two straight lines are usually used to approximate the magnetization curve (Fig. 1, a). In this case, the magnetic field induction in the interval from 0 to *H* is written as follows:

$$B = \mu_i H, \qquad (1)$$

$H > H_i$ and for the interval:

$$B = B_i + \mu'_i (H - H_i), \qquad (2)$$

Here H_i and B_i - strain and induction at the breaking point, respectively, $\mu_i = B_i / H_i$, $\mu'_i = (B_i - B_0) / H_i$, and the second curve $H = 0$ in $B = B_0$.

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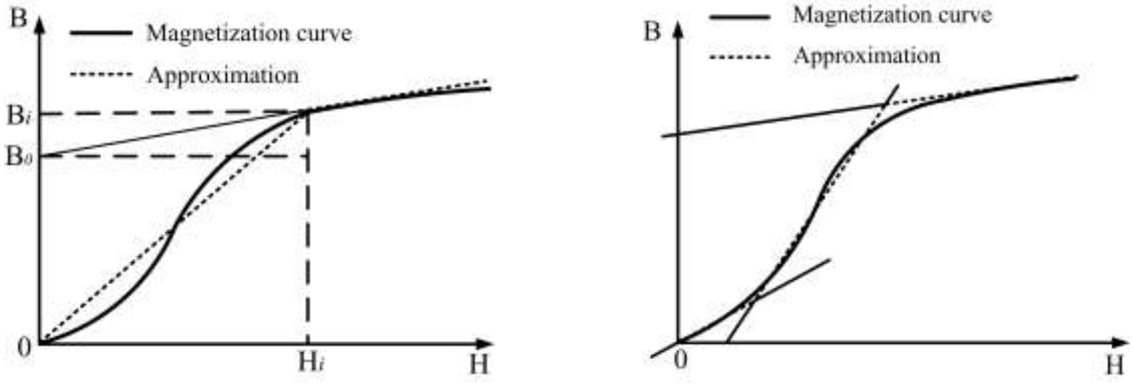

a) б)

Figure 1. Linear piecewise approximation.

In the approximation using two straight lines, the magnetization curve is described with a small accuracy. An increase in the number of straight lines leads to a significantly more accurate representation of the magnetization curve.

If a ferromagnetic material is magnetized from a demagnetized state to a magnetic saturation state, no less than three straight lines are used to construct the curve (Fig. 1, b) [1]. In this case, the tangent of the slope angle of the first straight line is set equal to the value of the initial magnetic susceptibility.

The advantage of the linear-piecewise approximation method is that nonlinear problems are replaced and solved by linear equations. The main drawback of this method is the increase in the calculation error due to the jump change occurring when the curve moves from one section to another section. The maximum deviation error of the approximating curve can range from 3% to 25%, depending on the number of sections.

Approximation using hyperbolic function [1, 6]. To approximate the near-saturation zone of the main magnetization curve, the Frelix formula in the form of a hyperbolic function is used (Fig. 2):

$$B = \frac{H}{k_1 + k_2 H}, \qquad (3)$$

where k_1 and k_2 are approximation coefficients determined using the selected point method or MathCAD software.

However, since the magnetization curve determined using this function is symmetrical with respect to the coordinate origin, it can be used in strong field zones where the signs of magnetic field strength and induction do not change.

The main disadvantages of the approximation using the hyperbolic function are that it cannot be used in the calculation of alternating current magnetic circuits and that the magnetic absorption does not have an extremum. This shows that the curve generated using this function does not match the experimental curve very well.

Approximation using the arctangent function. Using this function, the following expression is used to create a mathematical model of the magnetization curve (Fig. 3):

$$B = k_1 arctg(k_2 H) + k_3 H, \qquad (4)$$

Increasing the value of the expression in the first term causes the graph of the function to asymptotically approach a straight line parallel to and away from the abscissa axis. The second component of the expression describes the change of the magnetic induction in this zone.

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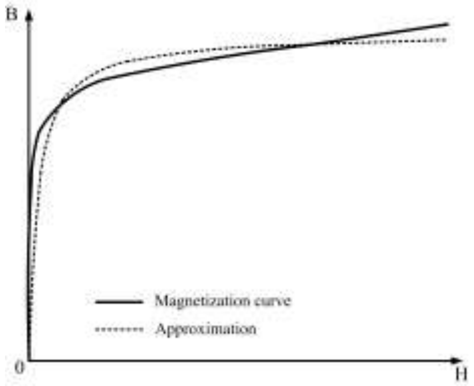

Figure 2. Approximation using a hyperbolic function.

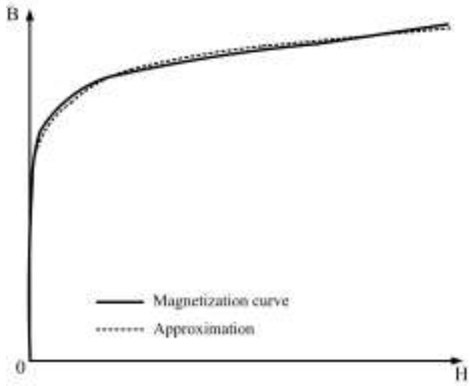

Figure 3. Approximation using a arctangent function.

Since this approximating function is odd, it can be used to calculate magnetic circuits with constant and alternating fields. When the magnetic field strength has a small value, the calculated curve is located above the actual magnetization curve, and when it has a large value, it is much lower than the actual magnetization curve.

k_1, k_2, k_3 - coefficients are determined by selecting three points from the magnetization curve. In this case, to determine the coefficient, it is required to solve the following equation:

$$\frac{B_1H_2 - B_2H_1}{B_3H_2 - B_2H_3} = \frac{H_2 arctg(k_2H_1) - H_1 arctg(k_2H_2)}{H_2 arctg(k_2H_3) - H_3 arctg(k_2H_2)}.$$

k_1 and k_3 and the coefficients can be determined using the following expressions:

$$k_1 = \frac{B_1H_2 - B_2H_1}{H_2 arctg(k_2H_1) - H_1 arctg(k_2H_2)}, \quad k_3 = \frac{B_1 - k_1 arctg(k_2H_1)}{H_1}.$$

Exponential approximation. The following common expression is used for exponential approximation of magnetization curves (Fig. 4)

$$B = e^{\frac{H}{k_1+k_2H}} - 1, \qquad (5)$$

where

$$k_1 = \frac{H_1H_2 \ln\frac{B_2+1}{B_1+1}}{(H_2-H_1)\ln(B_1+1)\ln(B_2+1)}, \quad k_2 = \frac{H_2\ln(B_1+1) - H_1\ln(B_2+1)}{(H_2-H_1)\ln(B_1+1)\ln(B_2+1)}.$$

In order to minimize the deviation of the approximating curve from the experimental main magnetization curve, when choosing the points used to determine the approximation coefficients, one of them is required to be in the zone before the saturation of the ferromagnetic material of the magnetization curve, and the other is at the beginning of the saturation curve close to it.

The method of exponential approximation can be used only in the calculation of magnetic circuits with a constant field.

Approximation using the logarithmic function. When approximating the main magnetization curve using this function, the following expression is used (Fig. 5):

$$B = k_1\sqrt{\ln(k_2H + 1)} \qquad (6)$$

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In this expression, the number 1 is added so that the argument of the logarithmic function does not become zero when *H=0*, *B=0* . This leads to a decrease in the accuracy of the approximation of the magnetization curve generated by this function to the experimental curve in weak field zones. The method of approximation using the logarithmic function can be used to calculate magnetic

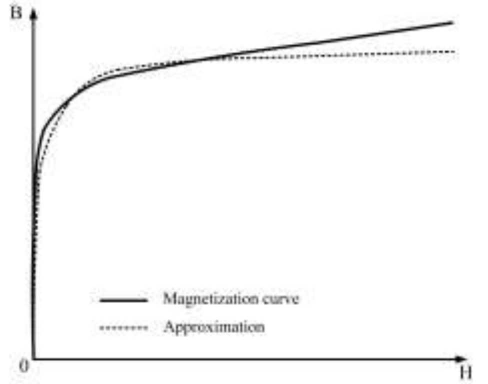

Figure 4. Approximation using exponential function

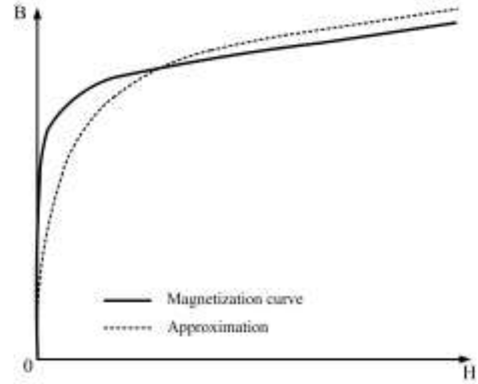

Fig. 5. Approximation using the logarithmic function

Approximation using a degree polynomial. It is possible to create a mathematical model with a high level of accuracy when approximating the main magnetization curve using an odd degree polynomial (Fig. 6). This method uses the following expression:

$$H = k_1 B + k_2 B^3 + k_3 B^5 + \dots,$$

or (7)

$$B = q_1 H + q_2 H^3 + q_3 H^5 + \dots,$$

where k_1, k_2, k_3,… …, q_1, q_2, q_3, … …- approximation coefficients determined using the selected point method or MathCAD software.

By increasing the number of terms on the right-hand side of the expression, the degree of agreement between the actual and calculated magnetization curves can be increased.
In most cases, it is limited to the use of two terms of the degree polynomial, that is:

$$H = k_1 B + k_2 B^3 \text{, ёки } B = q_1 H + q_2 H^3 \quad (8)$$

In this case, the calculated curve lies below the bending zone of the actual magnetization curve, and above it after the bending zone. When three terms of the degree polynomial are used, the calculated curve lies above the bend zone of the true magnetization curve and below the bend zone. In this way, an admissible approximation is achieved only in certain sections of the magnetization curve.

When using two terms of a degree polynomial, the approximation coefficients are determined as follows:

$$q_1 = \frac{B_1 H_2^3 - B_2 H_1^3}{H_1 H_2^3 - H_2 H_1^3}, \quad q_2 = \frac{B_2 H_1 - B_1 H_2}{H_1 H_2^3 - H_2 H_1^3}.$$

Approximation using a degree polynomial can be used to calculate magnetic circuits with a constant and variable field. It should be noted that the error does not exceed 4% when using the seventh degree polynomial when constructing the magnetization curve using this method. But as a result of the increase in the number of terms of the degree polynomial, difficulties may arise in determining the approximation coefficients. By reducing the degree of the polynomial, the number of coefficients can be reduced, but in this

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case the accuracy of the approximating curve decreases, and the maximum error compared to the nominal value is about 11%.

Approximation using hyperbolic sine. When approximating the main magnetization curve using this function, the following expression is used (Fig. 7):

$$H = k_1 sh(k_2 B). \qquad (9)$$

Approximation by this function is similar to approximation by a polynomial written in the degree of . By expanding the hyperbolic sine into a series, a polynomial written in terms of degree can be formed.

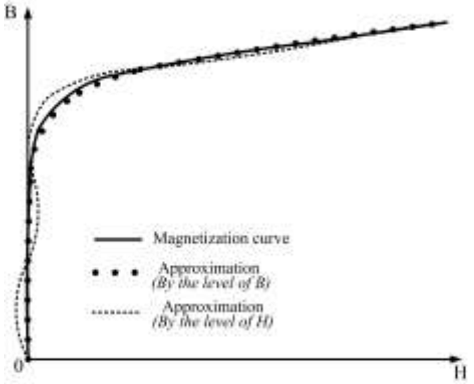

Figure 6. Approximation using a degree polynomial

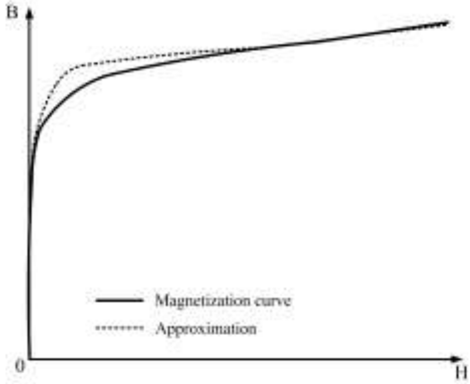

Figure 7. Approximation using hyperbolic sine

k_1 and k_2 coefficients are determined using the selected point method or MathCAD software and are expressed in the following form:

$$k_1 = \frac{H_2}{sh(k_2 B_2)}, \quad k_2 = \frac{\ln(\frac{H_2}{H_1})}{B_2 - B_1}.$$

The hyperbolic sine approximation method can be used to calculate constant and variable field magnetic circuits. The magnetization curve constructed using this method can deviate from the experimental curve by about 5%.

Approximation using hyperbolic tangent. Approximation using this function uses the following expressions (Figure 8):

$$H = k_1 th(k_2 B). \qquad (10)$$

$$B = q_1 th(q_2 H). \qquad (11)$$

- Approximation by this function is similar to approximation by a polynomial written in terms of the degree of *B* (or *H*). The curve produced by approximation by hyperbolic tangent is described with a larger error than the curve of magnetization approximated by hyperbolic sine. With the help of these functions, it is proposed to calculate magnetic circuits with a constant and variable field using the approximation method.
- Approximation using a spline function. The existence of a large number of experimental points and the spline method are used to approximate the magnetization curves of ferromagnetic materials with a high degree of accuracy. The main difficulty in spline approximation is the determination of its coefficients. MathCAD software is used to determine spline function coefficients (Figure 9). The spline function is studied in 4 groups:
- - cubic spline (cspline);
- - parabolic spline (pspline);
- - b-spline (bspline);чизиқли сплайн (*lspline*).

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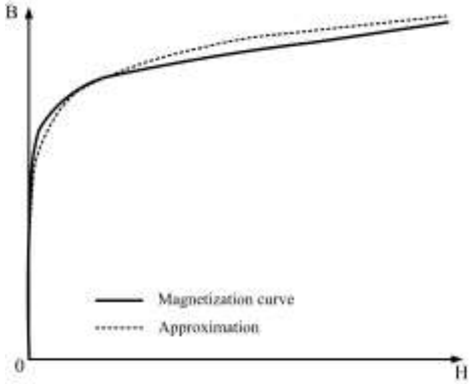

Figure 8. Approximation using hyperbolic tangent

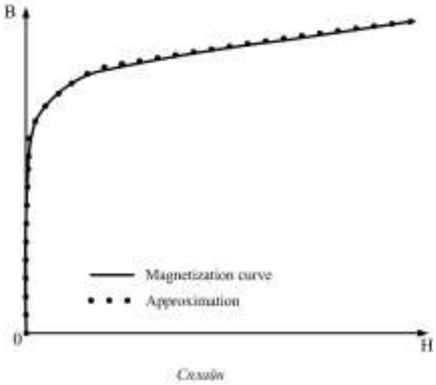

Figure 9. Approximation using a spline function

The magnetization curve approximated by the spline function can deviate from the nominal value by approximately 2% in the bending zone and by 1% over the entire section. Alternatively, by increasing the number of points, it is possible to reduce the deviation error by 0.1%. The main disadvantages of spline-function approximation are the lack of a general expression that fully forms the curve and the need to use special software tools to determine the coefficients.

Approximation using a rational function. The best result can be obtained by approximating the basic magnetization curve using this function:

$$B = \frac{k_i H^i + k_{i-1} H^{i-1} + ... + k_0}{q_i H^i + q_{i-1} H^{i-1} + ... + q_0}, \quad (12)$$

where p_i and q_i – coefficients, $i = 0...n.$

The upper coefficient of the degree polynomial in the fractional picture is equal to its value in the ordinate of the experimental points in the saturation section and indicates the asymptotic description of the function in this section. The minor coefficient of the degree polynomial is zero when the main magnetization curve passes through the coordinate origin. The remaining coefficients are determined using the method of selected points or MathCAD software. The following table lists the approximating functions, their expressions for determining the approximating coefficients, their error ranges, and their advantages and disadvantages.

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Approximation method and function	Expression for determining coefficients	Error of deviation from experiment, %	Benifits	Drawbacks
Linear-sectioning $B=\mu_i H$, $B = B_i + \mu'_i (H - H_i)$	$\mu_i = B_i / H_i, \quad \mu'_i = (B_i - B_0) / H_i$	3÷25	Nonlinear problems are reduced to linear equations.	Фақат магнит хоссасининг ночизиқлиқ даражаси паст бўлган материалларда қўллаш мумкин.
Hyperbolic function $B = \frac{H}{k_1 + k_2 H}$	$k_1 = \frac{H_1 H_2 (B_2 - B_1)}{B_1 B_2 (1 - H_1)}, \quad k_2 = \frac{1}{B_2} - \frac{k_1}{H_2}$	4÷6	It can be used in cases where a strong field occurs in the material.	Ўзгарувчан магнит майдонли занжирларини ҳисоблашда қўллаб бўлмайди. Магнит сингдирувчанлик экстремумга эга эмас.
Arctangent function $B = k_1 arctg(k_2 H) + k_3 H$	$k_1 = \frac{B_1 H_2 - B_2 H_1}{H_2 arctg(k_2 H_1) - H_1 arctg(k_2 H_2)}$, $k_3 = \frac{B_1 - k_1 arctg(k_2 H_1)}{H_1}$, $\frac{B_1 H_2 - B_2 H_1}{B_3 H_2 - B_2 H_3} = \frac{H_2 arctg(k_2 H_1) - H_1 arctg(k_2 H_2)}{H_2 arctg(k_2 H_3) - H_3 arctg(k_2 H_2)}$.	2÷5	It is used in cases where a constant or alternating magnetic field occurs in the material.	Магнит майдон кучланганлигининг қиймати катта бўлганда ҳосил қилинган эгри чизиқ экспериментал эгри чизиққа нисбатан анча пастда жойлашади.
Exponential function $B = e^{\frac{H}{k_1 + k_2 H}} - 1$	$k_1 = \frac{H_1 H_2 \ln \frac{B_2 + 1}{B_1 + 1}}{(H_2 - H_1) \ln(B_1 + 1) \ln(B_2 + 1)}$, $k_2 = \frac{H_2 \ln(B_1 + 1) - H_1 \ln(B_2 + 1)}{(H_2 - H_1) \ln(B_1 + 1) \ln(B_2 + 1)}$	3÷7	Approximation coefficients are easy to determine.	Материалда доимий магнит майдон юзага келадиган ҳолатларда қўлланилади.
The logarithm function $B = k_1 \sqrt{\ln(k_2 H + 1)}$	$k_1 = \frac{B_1}{\sqrt{\ln(\frac{H_1 e^{(B_1^2 - B_2^2)}}{H_1 - H_2}) + 1}}$, $k_1 = \frac{B_1}{\sqrt{\ln(\frac{H_1 e^{(B_1^2 - B_2^2)}}{H_1 - H_2}) + 1}}$,	5÷13		
A degree polynomial (seventh/fifth degree polynomial) $H = k_1 B + k_2 B^3 + k_3 B^5 + \ldots$ $B = q_1 H + q_2 H^3 + q_3 H^5 + \ldots$	$k_1 = \frac{H_1}{B_1} - \frac{B_2 H_1 B_1^2 - B_1^3 H_2}{B_1^3 B_2 - B_1 B_2^3}$, $k_2 = \frac{B_2 H_1 - B_1^3 H_2}{B_1^3 B_2 - B_1 B_2^3}$, $q_1 = \frac{B_1 H_2^3 - B_2 H_1^3}{H_1 H_2^3 - H_2 H_1^3}$, $q_2 = \frac{B_2 H_1 - B_1 H_2}{H_1 H_2^3 - H_2 H_1^3}$	(3÷4)/(5÷11)	It is used in cases where a constant or alternating magnetic field occurs in the material.	Даражали полином ҳадлари ортганда аппроксимацияла ш коэффициентлари ни аниқлашда қийинчиликлар юзага келади.

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Hyperbolic sine $H = k_1 sh(k_2 B)$	$k_1 = \frac{H_2}{sh(k_2 B_2)}, \quad k_2 = \frac{\ln(\frac{H_2}{H_1})}{B_2 - B_1}$	3÷5		Магнитланиш эгри чизиғи бошқа функцияларга нисбатан юқори хатоликда қурилади.
Hyperbolic tangent $H = k_1 th(k_2 B),$ $B = q_1 th(q_2 H)$	$k_1 = \frac{H_2}{th(k_2 B_2)}, \quad k_2 = \frac{\ln(\frac{H_2}{H_1})}{B_2 - B_1}, \quad q_1 = \frac{B_2}{th(k_2 H_2)},$ $q_2 = \frac{\ln(\frac{B_2}{B_1})}{H_2 - H_1}$	4÷7		
Spline is a function There is no common expression. MathCAD program cspline, pspline, bspline, lspline functions are used	$V_s = cspline(B_1, H_1),$ $H_2 = H_1,$ $H_3 = \text{int}\,erp(V_s, B_1, H_1, H_2),$ $f(x) = B_1(H_2)$	0.1÷2	It is used in the construction of the magnetization curve of any magnetic material with several experimental points.	MathCAD дастурида ишлаш бўйича кўникмага эга бўлиш лозим.
Rational function $B = \frac{k_i H^i + k_{i-1} H^{i-1} + ... + k_0}{q_i H^i + q_{i-1} H^{i-1} + ... + q_0}$	$k_0 = 0, \quad q_0 = 0.$ The remaining coefficients are determined using special functions of the Mathcad program.	0.1÷1	It is used in constructing the magnetization on curve of any magnetic material.	Аппроксимациялловчи коэффициентлар сонининг кўплиги ҳисобига уларни аниқлашда қийинчиликлар юзага келади.

Thus, when constructing the magnetization curve of the ferromagnetic material of distributed parameter devices, it is necessary to choose the method of approximating the magnetization curves based on the operating modes of this device and the conditions of their use, and taking into account the calculation capabilities of the control systems.

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