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The Nonlinear Magnetic Flux Diffusion in Superconductors

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Abstract

The problem of the penetration of a magnetic field into a high-temperature superconductor, which is in the regime of flux creep in an external magnetic field, is considered. Analytical formulas are obtained for the depth and rate of penetration of a magnetic field into a superconductor depending on the values of the problem parameter, namely, on the exponent n characterizing the rate of penetration of vortices into the superconducting half-space.

Key words : *flow dynamics, magnetic induction, self-similarity.*

Theoretical studies of the patterns of magnetic flux penetration in a various regimes of superconductors were carried out in classical works [1-3]. The dynamics of magnetic flux penetration under the assumption that the differential resistance does not depend on the magnetic field was studied in [2]. In this paper, we consider the nonlinear diffusion problem of the penetration of a magnetic flux into a superconductor taking into account the nonlinear current-voltage characteristic of superconductors, which is valid in the region of low eclectic fields and in the regime of flux creep. An exact numerical solution is obtained, describing the spatial and temporal evolution of the penetration of the current density, the magnetic and electric fields in the sample. To simulate the process of evolution of small perturbations of the electromagnetic field in space and time, we use the system of equations of macroscopic electrodynamics [3, 4]. The distribution of magnetic induction $\vec{B}$, electric field $\vec{E}$, and transport current in the superconductor are determined by the following equation

$$\operatorname{rot}\vec{B} = \mu_0 \vec{j}. \qquad \operatorname{rot}\vec{E} = \frac{d\vec{B}}{dt}. \tag{1}$$

Using the mathematical formalism developed in [2], we study the influence of differential resistance $\rho_f(B)$, on the process of penetration of the magnetic flux the viscous flow regime. The current-voltage characteristic in the regime of viscous flow of vortices can be written in the form

$$\vec{E}=\rho(B)\,\vec{j}. \tag{2}$$

Here $\vec{j} = j_c(\vec{B},T)$. Combining relation (1) with equation (2), we obtain a nonlinear diffusion equation for the magnetic flux induction $\vec{B}(\vec{r},t)$ in the following form

$$\frac{d\vec{B}}{dt} = \frac{1}{\mu_0}\nabla\left[\rho(B)\nabla\vec{B}\right]. \tag{3}$$

Obviously, the space-time structure of the solution of the diffusion equation (3) is determined by the nature of the dependence of the differential resistivity on the magnetic field induction B. Usually in a real experimental situation differential resistance $\rho(B)$ increases with increasing magnetic field induction

$$\rho(B) = \frac{\Phi_0}{\eta c^2}\vec{B} = \rho_n \frac{\vec{B}}{B_{C2}}, \tag{4}$$

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where ρ_n is the differential resistance in the normal state; n is the viscosity coefficient, B_{c2} is the upper critical field of the superconductor. In the case when the differential resistivity $\rho(B)$ is a linear function of the magnetic field induction B, the exact solution of the diffusion equation (3) can be easily obtained using known scaling methods [2]. For the complex dependence $\rho(B)$, one can use the empirical exponential dependence $r(B) \gg B^n$, where n is a positive constant parameter.

Let's consider the evolution of the magnetic flux injected in the infinite thin film (the xy plane) of a type-II superconductor (the flux lines are perpendicular to the surface). We assume the problem to be homogeneous along y, so the local magnetic induction B depends only on the coordinate x and on time. The current flows along y. An applied magnetic field is absent. For this dimensional geometry [5], the spatial and temporal evolution of the magnetic field induction is $B(r,t)$ described by the following nonlinear diffusion equation in a generalized dimensionless form

$$\frac{db}{d\tau} = \frac{d}{d\xi}\left(b^n\left[\frac{db}{d\xi}\right]^q\right), \qquad (5)$$

where we have introduced dimensionless parameters $b=\frac{B}{B_e}$, $\xi=\frac{\mu_0 j_0}{B_e}x$, $\tau=\frac{t}{t_0}$, $j=\frac{j}{j_0}$, $B_e=\mu_0 j_0 v_0 t_0$ and variables; $x_p = \frac{B_e}{\rho_0 j_c}$ - is the depth of penetration of the magnetic field in the Bean model; $t_0 = \rho_n \frac{j_c^2\mu_0}{B_e^2}$ is the diffusion time; q is a positive constant parameter. The diffusion equation (5) can be integrated analytically, taking into account the appropriate initial and boundary conditions at the center of the sample and at its edges. Let us consider the case when the magnetic field applied to the sample increases with time according to a power law with exponent $\alpha > 0$

$$b(0,t)=b_0(1+t)^{\alpha}. \qquad (6)$$

$$b(x_p,t)=0, \qquad (7)$$

The boundary condition (5) is equivalent to a linear increase in the magnetic field with time, which corresponds to the real experimental situation. It is easy to see that the case $\alpha=0$ describes a constant applied magnetic field on the surface of the sample, while the case $\alpha=1$ corresponds to a linearly increasing applied field, respectively. The spatial and temporal profiles of magnetic flux penetration into the sample depend on a set of three independent parameters, n, q, and α.

Here we consider the different cases, namely n = 0, 1, 2 and q = 0, 1. All examples are computed with N = 100 polynomials for the x and y-dependences, and Nt = 1000 time steps. Note that larger values of these parameters have only an effect on the solution below plotting accuracy, i.e., the resulting figures would be indistinguishable from the ones shown. We first consider the case n =0 and q = 1 in Fig. 1, on the left the initial condition, on the right the solution for t = 0.5. The solution is clearly unstable in the sense that the initial perturbations grow. In addition the simulations were performed for the coefficient α=1, final time t=10, time discretization M = 91, and space discretization N =82. The spatial and temporal profiles of the magnetic flux are shown in Figures 1-3.

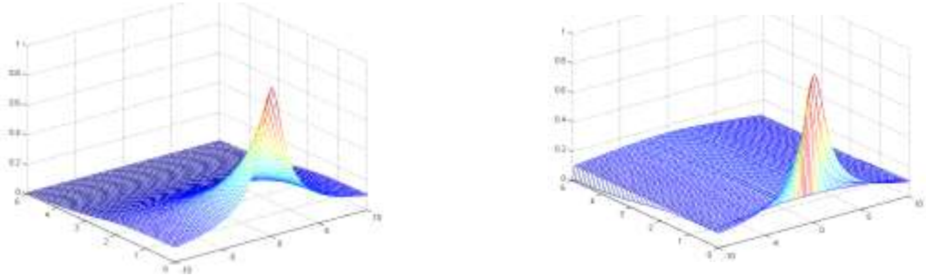

Fig.1. The effective magnetic flux penetration of the at n=0 and q=1 for t=0.5 and t=1.

Next, we consider the case n = 1 and q = 1 in Fig. 3, on the left the initial condition, on the right the solution for t = 0.5. The solution is clearly unstable in the sense that the initial perturbations grow.

The solution for the initial data (6) for n =1 and q =.1 at time t = 1 can be seen on the right of Fig. 2. The initial perturbations appear once more to grow in time. Schematically, the evolution of the process of penetration of the magnetic field in the regime of viscous flow of vortices with a power-law dependence $b(x,t)$ on the exponent n is shown in Figure 2.

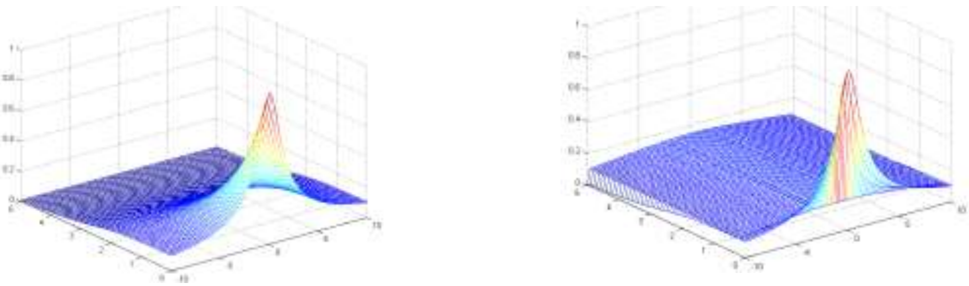

Fig.2. The effective magnetic flux penetration of the at n=1 and q=1 for t=1 and t=5.

Next we consider the case n = 2 and q = 1 in Fig. 3, on the left the initial condition, on the right the solution for t = 5. The solution is clearly unstable in the sense that the initial perturbations grow.

The solution for the initial data for n =2 and q =1 at time t = 5 can be seen on the right of Fig. 3. The initial perturbations appear once more to grow in time. Schematically, the evolution of the process of penetration of the magnetic field in the regime of viscous flow of vortices with a power-law dependence $b(x,t)$ on the exponent n is shown in Figure 3.

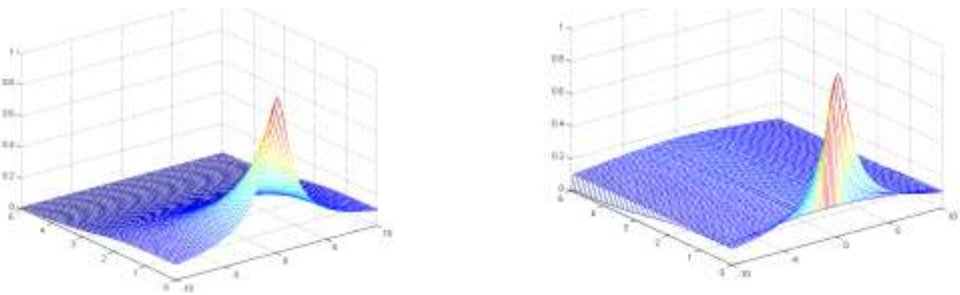

Fig.3. The effective magnetic flux penetration of the at n=2 and q=1 for t=5 and t=10.

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The final distribution reached in the experiment exhibits a typical shape characteristic for diffusion processes with diffusion coefficient which depends exponentially on the concentration.

Conclusion

In summary, we have considered problem of nonlinear diffusion of the magnetic flux injected in an infinite thin type-II superconductor. We have solved it numerically in the most interesting case of flux flow resistivity proportional to the power-law dependence of local magnetic induction B. The obtained flux space-time distributions are of the self-similar form with rather striking scaling functions.

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