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On Generalized Derivations Of Jordon Algebras

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Annotation. In the present article the concept of generalized derivations of Jordan algebras is introduced and the general properties of generalized derivations are studied. In particular, invariants are constructed in Jordan algebras using generalized derivations. Also, a description, for some values of the parameters α, β, γ, of the sets of all (α, β, γ) -derivations of Jordan algebrasis given.

Key words: Vector space, Jordon algebra, linear map, derivation, generalized derivation.

1-olimiting. Jordanov's algebra is - I satisfy Jitsconditions

$$a{\cdot}b = b{\cdot}a, a, b \in J,$$

$$(a^2{\cdot}b){\cdot}a = a^2{\cdot}(b{\cdot}a), a, b \in J.$$

$J \times J \to J -$ linear reflection of vector space over a field V F

The definition - differentiation of Jordan algebras (α, β, γ)can be written as follows:

*2-o limiting***.** Let Jordanov's algebra. $(J, \cdot)-$

A linear operator is called$d \in End(J) J(\alpha, \beta, \gamma)-$ a differentiation if for all elements there are such , $x, y \in J \alpha, \beta, \gamma \in$**C**which fulfills the following condition

$$\alpha d(x{\cdot}y) = \beta(d(x){\cdot}y) + \gamma(x{\cdot}d(y)).$$

Sets of all jordan differentiations$(\alpha, \beta, \gamma)-J$yx algebras denote by $Der_{(\alpha,\beta,\gamma)}(J)$. This set is determined by the following:

$$Der_{(\alpha,\beta,\gamma)}(J) = \{d \in End(J): \alpha d(x{\cdot}y) = \beta(d(x){\cdot}y) + \gamma(x{\cdot}d(y)), x, y \in J\}.$$

Where the set is a subspace of the vector space $Der_{(\alpha,\beta,\gamma)}(J) End(J)$.

Theorem. Let be a Jordanian algebra with a unit$J-$ ym element and $\alpha, \beta, \gamma \in$ **F.** Then the vector space has the following$Der_{(\alpha,\beta,\gamma)}(J)$formy.

1. , $Der_{(1,1,1)}(J) = Der(J)$where the Lie algebra of all differentiations of Jordan algebras$Der(J)$–y ; J

2. Sets and$Der_{(1,1,0)}(J) \subseteq End(L) Der_{(1,1,0)}(J)$

it is possibleto identically equalize all the idempotent elements of the Jordan algebra $Id(J)$

$$p \in Id(J), \qquad d(x) = p{\cdot}x, \ \ x \in J$$

3. $Der_{(1,1,-1)}(J) \subseteq Der_{(1,0,0)}(J) \equiv 0.$

4. .$Der_{(0,1,1)}(J) \equiv 0$

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5. The set can be identically equal$Der_{(0,1,-1)}(J)$to the set $Z_o(J) = \{a \in J \mid (b \cdot a) \cdot c = b \cdot (a \cdot c),\ \ b, c \in J\}$

$$a \in Z_o(J),\ \ d(x) = a \cdot x,\ \ x \in J.$$

6. 0;$Der_{(0,1,0)}(J) \equiv$

7. Esli $\delta \neq 1$, then$Der_{(\delta,1,0)}(J) \equiv 0$.

*Proof***.** From equality $d(x \cdot y) = d(x) \cdot y, x, y \in J$ we get . $d(x) = d(e) \cdot x,\ \ x \in J$Then, since Zhordanov's algebra is commutative, J

$$d(x \cdot y) = d(e) \cdot (x \cdot y) = d(x) \cdot y = d(y) \cdot x, \quad x, y \in J$$

and

$$d(x \cdot y) = d(x \cdot e) \cdot y = (d(e) \cdot x) \cdot y = (d(e) \cdot y) \cdot x, \quad x, y \in J.$$

Hence$d(e)$–the central element. As well as

$$d(e) = d(e \cdot e) = d(e) \cdot d(e),$$

i.e. the element is idempotent$d(e)$yy element.

Conversely , for each central idempotent element of the Jordan algebras$p$y J

Reflection belongs to the set and $d(x) = p \cdot x,\ \ x \in J, Der_{(1,1,0)}(J)$

va therefore, the sets can be identically aligned with the multiplicity of all the idempotent elements of the Jordan algebra .$Der_{(1,1,0)}(J) J$

3. By definition $Der_{(1,1,-1)}(J) = \{d \in End(J) \mid d(x \cdot y) = d(x) \cdot y - x \cdot d(y),\ x, y \in J\}$. There is a followingequality:

$$d(x) = d(e) \cdot x - d(x), \qquad x, y \in J,$$

$$d(x \cdot y) = \big(d(e) \cdot x - d(x)\big) \cdot y - x \cdot \big(d(e) \cdot y - d(y)\big) =$$

$$(d(e) \cdot x) \cdot y - d(x) \cdot y - x \cdot (d(e) \cdot y) + x \cdot d(y) =$$

$$(d(e) \cdot x) \cdot y - x \cdot (d(e) \cdot y) - d(x \cdot y), \qquad x, y \in J,$$

$$d(e) = d(e \cdot e) = d(e) \cdot e - e \cdot d(e) = 0.$$

Hence

$$d(x \cdot y) = -d(x \cdot y) = 0,\ \ x, y \in J$$

and

$$d(e \cdot x) = -d(e \cdot x) = d(x) = 0,\ \ x \in J.$$

From the Last Equality $Der_{(1,1,-1)}(J) \equiv 0$.

4. By definition, the set is defined in the following$Der_{(0,1,1)}(J)$way:

$Der_{(0,1,1)}(J) = \{d \in End(J) \mid d(x) \cdot y = -x \cdot d(y), x, y \in J\}$.

Then $d(e)\cdot x = -e\cdot d(x) = -d(x),\ x\in J$, , . $d(e)\cdot e = -e\cdot d(e)d(e) = 0$

Hence

$$d(x) = e\cdot d(x) = -d(e)\cdot x = 0, \qquad x\in J.$$

So, $Der_{(0,1,1)}(J)\equiv 0$.

5. As iny she, for the set you can get equals $Der_{(0,1,-1)}(J) = \{d\in End(J) \mid d(x)\cdot y = x\cdot d(y), x, y\in J\}$

$$d(e)\cdot x = e\cdot d(x) = d(x),\ \ x\in J,$$

$$d(x)\cdot y = (d(e)\cdot x)\cdot y = \big(x\cdot d(e)\big)\cdot y = x\cdot d(y) = x\cdot(d(e)\cdot y), x, y\in J.$$

Now from the set

$$Z_o(J) = \{a\in J \mid (b\cdot a)\cdot c = b\cdot(a\cdot c),\ \ b, c\in J\}$$

let's take arbitrarilyyy element$a\in Z_o(J)$ z. Let's show that reflection$d(x) = a\cdot x,\ \ x\in J$.

Belongs to the set $Der_{(0,1,-1)}(J)$.

$$d(x)\cdot y = (a\cdot x)\cdot y = (x\cdot a)\cdot y = x\cdot(a\cdot y) = x\cdot d(y),\ \ x, y\in J.$$

6. By definition, a set is defined by following$Der_{(0,1,0)}(J)$it:

$Der_{(0,1,0)}(J) = \{d\in End(J) \mid d(x)\cdot y = 0, x, y\in J\}$.

Thereis a followingequality:

$$d(x) = d(x)\cdot e = 0, \qquad d(e) = d(e)\cdot e = 0, \qquad x\in J.$$

Hence$Der_{(0,1,0)}(J)\equiv 0$.

7. And at the end , of the set

$$Der_{(\delta,1,0)}(J) = \{d\in End(J) \mid \delta d(x\cdot y) = d(x)\cdot y,\ \ x, y\in J\}$$

you can get equality

$$\delta d(x) = \delta d(x\cdot e) = d(x)\cdot e = d(x),\ \ x\in J$$

$$\delta d(x) = d(x), \qquad (\delta - 1)d(x) = 0, \delta\neq 1, d(x) = 0 \quad x\in J.$$

Since

$$0 = (\delta - 1)^{-1}0 = (\delta - 1)^{-1}(\delta - 1)d(x) = d(x).$$

So, $Der_{(\delta,1,0)}(J)\equiv 0$.

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