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Relationship of Quaternions and Vector Algebra

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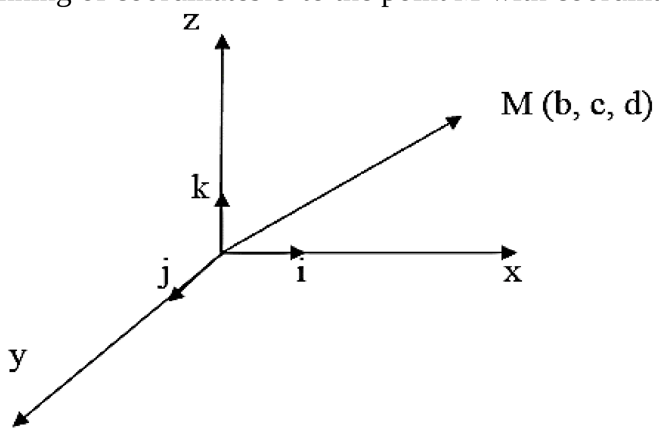
Annotation: In this article applications of quaternion theory to vector algebra are highlighted. The relations between operations on three-dimensional space vectors and calculations on quaternions are given.

Key words: theory of quaternions, algebra of vectors, quaternion, scalar multiplication, vector multiplication, length of vector.

In the middle of the 19th century the appearance of quaternions stimulated various researches in the fields of mathematics and physics. In particular, because of quaternions, one of the important branches of mathematics - algebra of vectors was created. It can be clearly noticed that the algebra of vectors arose after preliminary conclusions of the theory of quaternions. The scientific works of the English mathematician W. Hamilton, who was really the founder of the theory of quaternions, dates back to the 50s of the 19th century, and the works of the American physicist and mathematician D. Gibbs to form the main place of vector algebra in mathematics dates back to the 80s of the 19th century.

Quaternion - (from latin quaterni - "of four") term was proposed by the English scientist Hamilton (1843) [1]. Below are represented the relationships between calculations on quaternions and operations on three-dimensional space vectors.

Let's recall some concepts known to us from geometry. If we take a right-angled coordinate system in space, and define vectors *i, j, k*, directed along the coordinate axis from the origin of the coordinates, and whose length is equal to 1 (picture 1), then any sum of the form $bi+cj+dk$ represents a certain vector. This vector is a vector from the beginning of coordinates *O* to the point *M* with coordinates *b, c, d* .



1 .

Any quaternion of the form $q=a+ bi+cj+dk$ represents vector of the form $bi+cj+dk$ and *a* a real number. We call the number *a the* numerical (real) part of the quaternion *q* , *and the expression* $bi+cj+dk$ is its vector part. Now let's look at two quaternions $q_1=a_1+b_1i+c_1j+d_1k$ and $q_2=a_2+b_2i+c_2j+d_2k$.

According to the rule of multiplying quaternions we get the following result [3] by multiplying them:

$$q_1q_2= -(b_1b_2+c_1c_2+d_1d_2) +(c_1d_2-d_1c_2)i+(d_1b_2-b_1d_2)j +(b_1c_2-c_1b_2)k \quad (1)$$

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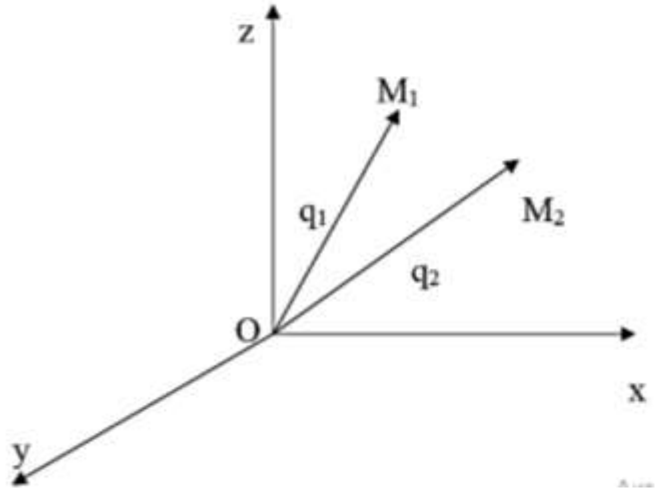
We write the numerical part and the vector parts of the quaternion $q_1 q_2$ separately :

numerical part of $q_1 q_2$ - $-(b_1 b_2 + c_1 c_2 + d_1 d_2)$ (2)

vector part of $q_1 q_2$ - $(c_1 d_2 - d_1 c_2)i + (d_1 b_2 - b_1 d_2)j + (b_1 c_2 - c_1 b_2)k$ (3)

Each of the expressions (2), (3) expresses a certain geometric meaning. We show that the sum of $b_1 b_2 + c_1 c_2 + d_1 d_2$ is equal to the expression $|q_1||q_2|\cos\varphi$ which determines multiplication of the modules of q_1 and q_2 quaternions and the cosine of the angle between them. Let's consider the scalar product of q_1, q_2 vectors. Note that the scalar product is denoted as (q_1, q_2), not a vector, but a certain number. Thus, according to the definition of the scalar product $(q_1,q_2) = |q_1||q_2|\cos\varphi$.

Let's prove the formula $(q_1,q_2) = b_1b_2+c_1c_2+d_1d_2$ (4). In picture 2 a triangle constructed on q_1 and q_2 of vectors is depicted. The first tip of the triangle lies at the origin of the coordinates and the other two tips are on the points M_1 and M_2 with coordinates b_1, c_1, d_1 and b_2, c_2, d_2.



Picture 2.

(M_1 and M_2 *points* are the tips of the q_1 and q_2 vectors respectively.)

The following are known to us:

$OM_1^2=b_1^2+c_1^2+d_1^2$, $OM_2^2=b_2^2+c_2^2+d_2^2$, $M_1M_2^2 = (b_1-b_2)^2+(c_1-c_2)^2+(d_1-d_2)^2$, after that $M_1M_2^2 = OM_1^2+ OM_2^2-(b_1b_2+c_1c_2+d_1d_2)$. And from the theorem of cosines: $M_1M_2^2 = OM_1^2+ OM_2^2-2\, OM_1 \cdot OM_2 \cdot \cos\varphi$ where φ - q_1 and q_2 the angle between the vectors.

By equating the above expressions, we get the following expression to be proved: $OM_1 \cdot OM_2 \cdot \cos\varphi=b_1b_2+c_1c_2+d_1d_2$.

Thus , the real part of the product of q_1 and q_2 quaternions is equal to the obtained with the opposite sign of scalar multiplication of q_1 and q_2.

If q_1 and q_2 if the vectors are perpendicular, it is clear that their scalar product is equal to zero ($\varphi=\pi/2$, $\cos\varphi=0$), and therefore the real part of the product $q_1 q_2$ is also zero. In this case q_1, q_2 will consist of clean vectors. The converse of this statement is also valid, that is, if q_1, q_2 is a clean vector, then the scalar product of q_1 and q_2 *is* equal to 0. q_1 and q_2 are perpendicular while $q_1 q_2 =-q_1q_2$ can also be seen from formula (1).

Expressing the geometric meaning of the vector part of the product q_1q_2 (the expression on the right side of equation (3)) is somewhat difficult. This expression is called the vector product of the vectors q_1 and q_2 and is denoted by $[q_1 ,q_2]$.

$[q_1,q_2] =(c_1d_2-d_1c_2)i+(d_1b_2-b_1d_2)j +(b_1c_2-c_1b_2)k$.

The vector $[q_1 ,q_2]$ *is* perpendicular to each of the vectors q_1 and q_2 , and its length is $|q_1||q_2|\sin\varphi$ or the face of the parallelogram S constructed from the vectors q_1 and q_2.

To prove the perpendicularity of the vectors $[q_1,q_2]$ and q_1 , as is well known, it suffices to check that the real part of the product of quaternions is equal to zero or that their product is a "clean" vector. But since $[q_1,q_2] =q_1q_2 +(q_1,q_2)$ from (1) and (4), then $q_1[q_1,q_2]=q_1(q_1q_2+(q_1,q_2)) =q_1^2q_2+ (q_1,q_2)\, q_1 = -|q_1|^2q_2 +(q_1,q_2)q_1$.

On the right side, a vector is formed, consisting of the sum of two more vectors. The perpendicularity of the vectors $[q_1 ,q_2]$ and q_2 can be shown in the same way.

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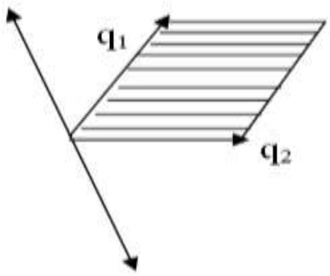
Now let's find the length of the vector $[q_1 ,q_2]$.

Its square is equal to $(c_1d_2-d_1c_2)^2+(d_1b_2-b_1d_2)^2+(b_1c_2-c_1b_2)^2$ or $(b_1^2+c_1^2+d_1^2)(b_2^2+c_2^2+d_2^2)-(b_1b_2+c_1c_2+d_1d_2)^2$. The last expression represents $|q_1|^2|q_2|^2-(q_1,q_2)^2$, or by the definition of scalar product $|q_1|^2|q_2|^2- |q_1|^2|q_2|^2cos^2\varphi$, or $|q_1|^2|q_2|^2sin^2\varphi$. Therefore, the square of the length of the vector $[q_1 ,q_2]$ is equal to $|q_1|^2|q_2|^2sin^2\varphi$, or S^2 , which is the statement required to be proved. The indicated properties of the vector $[q_1 ,q_2]$, i.e., the perpendicularity of q_1 and q_2 and its length equal to S, do not completely define it. $[q1,q2]$, q_1 and q_2 and its length equal to S, do not completely determine it. Such properties are represented by two mutually opposite vectors. (picture 3).



Picture 3.

Thus, for pure vector quaternions, the formula $q_1q_2 = - (q_1, q_2)+ [q_1 ,q_2]$ is appropriate. Here, (q_1, q_2) *is* the scalar product of q_1,q_2 and $[q_1,q_2]$ is the vector product. It can be seen from them that scalar and vector multiplications are "pieces" of quaternion multiplication. The operations of vector scalar multiplication and vector multiplication belong to the section of vector algebra of mathematics that has interdisciplinary applications in physics, mathematics itself, especially in mechanics.

Quaternion multiplication is a key tool in solving some problems of geometry and mechanics because it combines 2 different multiplications of vectors, namely scalar and vector multiplication.

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