On The Stability of the Approximate Solution of the Galerkin Method for A Parabolic Boundary Problem with Divergent Main Part

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Annotation. The article considers the application of the Galerkin method to the solution of boundary value problems of parabolic type with a divergent main part, when the boundary condition contains the time derivative of the desired function.

Such non-classical problems with boundary conditions, containing the time derivative of the desired function, arise when studying a number of applied problems, for example, when a homogeneous isotropic body is placed in an inductor of an induction furnace and an electromagnetic wave falls on its surface. Such problems are little studied, so the study of problems of parabolic type, when the boundary condition contains the time derivative of the desired function, is relevant. In this paper, we study a generalized solution of the problem under consideration in the space $\widetilde{H^{1,1}}(Q_T)$.. The proposed boundary value problem is considered under certain conditions for the function involved in the equation and the boundary condition, which allow the existence and uniqueness of the generalized solution. For the numerical solution of the problem under consideration, an approximate solution was constructed using the Galerkin method. The concept of stability of the Galerkin process for this problem is introduced. The purpose of the study is to obtain a condition for the stability of the computational process of the considered mixed problem. Using the Galerkin method under consideration, the problem is reduced to solving a system of ordinary differential equations. Next, we consider the "perturbed" problem for the system of the Galerkin method and obtain estimates for the difference between the solutions of the original and perturbed systems. The article establishes the stability of the approximate solution of the Galerkin method in the space $L_2(0,\mathrm{T},\mathrm{H}^1)$ of the problem under consideration, under conditions of strong minimality of the coordinate system.

Keywords: Boundary value problem, quasilinear equation, boundary condition, Galerkin method, generalized solution, parabolic problem, approximate solution, error estimate, monotonicity, inequalities, time derivative, boundary, region, scalar product, norm, continuity, desired function.

Introduction. When studying a number of topical technical problems, it becomes necessary to study mixed problems of the parabolic type, when the boundary condition contains the time derivative of the desired function. Problems of this type arise, for example, when a homogeneous isotropic body is placed in an inductor of an induction furnace and an electromagnetic wave falls on its surface. Some nonlinear problems of parabolic type with a boundary condition containing the time derivative of the desired function were considered, for example, in [1-3]. Many scientists were involved in the construction of an approximate solution using the Galerkin method and obtaining a priori estimates of the approximate solution for parabolic quasilinear problems without a time derivative in the boundary condition: Mikhlin

S.G., Douglas J. Jr, Dupont T. , Dench J. E., Jr, Jutchell L. and others [4-9]. And quasi-linear problems, when the boundary condition contains the time derivative of the desired function using the Galerkin method, have been little studied [10-15]. In [16], the stability of the approximate solution of the Galerkin method of the problem under consideration in the space $L_\infty(0, \mathrm{T}, \hat{L}_2(\Omega))$ was established.

Formulation of the problem. In this paper, we consider a quasilinear problem of parabolic type, when the boundary condition contains the time derivative of the desired function:

$$\begin{cases} u_t - \frac{d}{dx_i} a_i(x,t,u,\nabla u) + a(x,t,u,\nabla u) = 0 \quad , \\ a_0 u_t + a_i(x,t,u,\nabla u)\cos(\nu, x_i) = g(x,t,u), \quad (x,t) \in S_t \ , \\ u(x,0) = u_0(x) \ , \ x \in \Omega \end{cases} \qquad (1)$$

где $\Omega - bounded\ domain\ in\ \mathrm{E}_m, m =$ dim $-$ dimension of domain Ω, $\quad Q_T = \{\Omega \times [0,T]\}$,

$$S_T = \{\partial\Omega \times [0,T]\}, \qquad a_0 = const > 0$$

Definition. A generalized solution from the space $\widetilde{\mathrm{H}^{1,1}}(Q_T) = \{u \in H^{1,1}(Q_T): a_0 u_t \in L_2(S_T)\}$ of problem (1) is a function from (H^1,1)̃(Q_T). satisfying the identity $\widetilde{\mathrm{H}^{1,1}}(Q_T)$

$$\int_{Q_T} \left(u_t\eta + a_i(x,t,u,\nabla u)\eta_{x_i} + a(x,t,u,\nabla u)\eta\right)dxdt + \int_{S_T} \left(\left(a_0 u_t + g(x,t,u)\right)\eta\right)dxdt = 0 \qquad (2)$$

$$\forall\, \eta \in H^{1,1}(Q_T)$$

Main results: Consider problem (1) under the following conditions

1) for $(x,t) \in \bar{Q}_T$ and arbitrary u, v, p and q inequalities

$$|a_i(x,t,u,p) - a_i(x,t,u,q)| \le \tilde{\mu}(|u|)|p-q|$$
$$|a_i(x,t,u,p) - a_i(x,t,v,p)| \le \widetilde{L_0}(|u|+|v|)|u-v|$$
$$|a(x,t,u,p) - a(x,t,u,q)| \le [\widetilde{L_1}(|u|) + L_1(|p|^\beta + |q|^\beta)]|p-q| \qquad (3)$$
$$|a(x,t,u,p) - a(x,t,v,p)| \le [\widetilde{L_2}(|u|) + |v|) + L_2(|p|^{2\beta})]|u-v|$$
$$|g(x,t,u) - g(x,t,v)| \le \widetilde{L_3}(|u|+|v|)|u-v|,$$

where $\tilde{\mu}(\tau)$, $\widetilde{L_\iota}(\tau)$ are continuous positive functions of $\tau \ge 0$.
L_1, L_2 $-$ positive constants.

2) The boundary S of the domain Ω is such that the inequalities [17-18]

$$\|u\|_{L\tilde{q}(\Omega)} \le \varepsilon\, \|\nabla u\|^2_{L_2(\Omega)} + C_\varepsilon \|u\|^2_{L_2(\Omega)},$$
$$\tilde{q} = \frac{2l}{l-2}\ , l > m; \qquad (4)$$
$$\|u\|_{L\bar{q}(S)} \le \varepsilon\, \|\nabla u\|^2_{L_2(\Omega)} + C_\varepsilon \|u\|^2_{L_2(\Omega)}\ , \qquad (5)$$
$$\bar{q} < \frac{2(m-1)}{m-2}.$$

3) Monotonicity condition. For any functions $u, v \le H^1$, the following inequality holds:

$$(a_i(x,t,u,\nabla u) - a_i(x,t,v,\nabla v), u_{x_i} - v_{x_i})_\Omega + (a(x,t,u,\nabla u) - a(x,t,v,\nabla v), u-v)_\Omega \ge 0 \qquad (6)$$

4) For $(x,t,u) \in \{\bar{\Omega} \times [0,T] \times E_1\}$ the function $g(x,t,u)$ is measurable in (x,t,u), is continuous in (t,u) and satisfies the inequality:

$$|g(x,t,u) - g(x,t,v)| \le g_0|u-v|, \qquad g(x,t,0) \in L_2(S_T) \qquad (7)$$

Let us construct an approximate solution using the Galerkin method. Let us take a coordinate system from the space $H^1(\Omega)$. Approximate solution U(x,t) will be sought in the form [19-25]:

$$U(x,t) = \sum_{k=1}^n C_k^n(t)\varphi_k(x) \qquad (8)$$

where $\mathrm{C}_k^n(t)$ are determined from the system of ordinary differential equations

ISSN NO: 2770-4491
Date of Publication:28-11-2022

$$(U_t,\varphi_j)_{\hat{L}_2}+(a_i(x,t,U,\nabla U),\varphi_{jx_i})_\Omega+(a(x,t,U,\nabla U),\varphi_j)_\Omega= = (g(x,t,U),\varphi_j)_S\ ,\quad j=\overline{1,n} \qquad (9)$$

with initial conditions

$$(U(x,O)-u_0,\varphi_j)_{H^1(\Omega)}=0$$

Here $\hat{L}_2(\Omega)$ is the function space with inner product

$$(u,v)_{\hat{L}_2}=(u,v)_\Omega+a_0(u,v)_s$$

Let us assume that the coordinate system $\{\varphi_k\}\subset H^1(\Omega)$ is strongly minimal in the space $\hat{L}_2(\Omega)$, i.e. there exists a constant q independent of n such that $0\le q\le {q_i}^n$, where ${q_i}^n$ are eigenvalues of the matrix $Q_n=\left\{(\varphi_k,\varphi_j)_{\hat{L}_2}\right\}_{k,j=1}^n$

Write system (9) in vector form

$$\begin{cases} Q_n\,\dot{C}_n(t)+R_n(t,C_n(t))=f_n(t,C_n(t))+g_n(t,C_n(t)) \\ \tilde{Q}_n\,C_n(0)=\mathrm{T}_n \end{cases} \qquad (10)$$

here

$R_n(t,C_n)=\left\{(a_i(x,t,U,\nabla U),\varphi_{kx_i})_\Omega\ \right\}_{k=1}^n,$

$f_n(t,C_n)=\{(a(x,t,U,\nabla U),\varphi_k)_\Omega\}_{k=1}^n,$

$g_n(t,C_n)=\{(g(x,t,U),\varphi_k)_S\}_{k=1}^n,\qquad \tilde{Q}_n=\left\{(\varphi_i,\varphi_k)_{H^1(\Omega)}\ \right\}_{i,k=1}^n,\ \mathrm{T}_n=\left\{(u_0,\varphi_k)_{H^1(\Omega)}\ \right\}_{k=1}^n,\quad C_n=\{C_n^k\}_{k=1}^n$

n-dimensional vectors.

Let us assume that instead of the Galerkin system (10) we solve the “perturbed” system

$$\begin{cases} (Q_n+\Gamma_n)\dot{\tilde{C}_n}(t)+R_n\left(t,\tilde{C}_n(t)\right)=g_n\left(t,\tilde{C}_n(t)\right)+f_n\left(t,\tilde{C}_n(t)\right)+\delta_n(t,C_n), \\ (\tilde{Q}_n+\Gamma_n^0)\,\tilde{C}_n(0)=\mathrm{T}_n+\Delta_n \end{cases} \qquad (11)$$

where $\tilde{C}_n(t)$ is the solution of the perturbed problem.

Definition. The Galerkin process for problem (10) is called stable if there exist positive constants $p_i\ \ (i=\overline{0,3})$ independent of n such that for sufficiently small matrix norms $\|\Gamma_n^0\|\ \ \|\Gamma_n\|$ and norms vectors $\|\delta_n(t,C_n)\|_{L_2(0,t,E_n)}$, $\|\Delta_n\|_{E_n}$

the inequality

$$\left\|\tilde{C}_n(t)-C_n(t)\right\|_{E_n}\le p_0\|\Delta_n\|_{E_n}+p_1\|\Gamma_n^0\|+p_2\|\Gamma_n\|+ +p_3\max_{\|C_n\|\le K}\|\delta_n(t,C_n)\|_{L_2(0,t,E_n)} \qquad (12)$$

An approximate solution $U(x,t)$ is called stable in the space $\hat{L}_2(\Omega)$ if an inequality similar to (12) holds for the difference

$\left\|\tilde{U}(x,t)-U(x,t)\right\|_{L_2}$, where $\tilde{U}(x,t)=\sum_{k=1}^n\tilde{C}_k^n(t)\varphi_k(x)$

Using the strong minimality of the coordinate system $\{\varphi_k\}$ in $\hat{L}_2(\Omega)$, we obtain the inequalities

$$\|C_n(t)\|_{E_n}^2\le\frac{1}{q}\|U\|_{\hat{L}_2}^2\le N/q \qquad (13)$$

$$\int_0^t\left\|\frac{d\mathrm{C}_n(t)}{dt}\right\|_{E_n}^2dt\le\frac{1}{q}\left\|\frac{dU}{dt}\right\|_{L_2(0,t,\hat{L}_2)}^2\le N/q$$

Then, in inequality (13) we set $K=N/q$.

Let the allowed errors $\Gamma_n,\ \delta_n(t,C_n)$ be such that

$$\|\Gamma_n\|\le\alpha q, \qquad (14)$$

in the ball $\|C_n(t)\|_{E_n}^2\le K$ the inequality

ISSN NO: 2770-4491
Date of Publication:28-11-2022

$\|\delta_n(t,C_n)\|^2_{L_2(0,t,E_n)} \le \delta C_к$ (15)

Here $z_n = \tilde{C}_n(t) - C_n(t)$.

From equation (11) we subtract equation (10). We multiply the resulting equation scalarly by the vector $z_n(t)$

$$\frac{1}{2}\frac{d}{dt}\left((Q_n+\Gamma_n)z_n,z_n\right)_{E_n} + \left(R_n(t,\tilde{C}_n)-R_n(t,C_n),z_n\right)_{E_n} + +\left(f_n(t,\tilde{C}_n)-f_n(t,C_n),z_n\right)_{E_n} = \left(g_n(t,\tilde{C}_n)-g_n(t,C_n),z_n\right)_{E_n} + (\Phi_n(t,C_n),z_n)_{E_n} \quad (16)$$

Denote

$$\Phi_n(t) = -\Gamma_n\dot{C}_n(t) + \delta_n(t,C_n)$$

Due to assumptions (4) - (7), $a_0 \ne 0$.

$$\left(R_n(t,\tilde{C}_n)-R_n(t,C_n),z_n\right)_{E_n} = ((a_i(\mathrm{x},\mathrm{t},\widetilde{U},\nabla\widetilde{U})-a_i(\mathrm{x},\mathrm{t},U,\nabla U),(\widetilde{U}-U)_{x_i})_\Omega$$
$$\ge v\|\nabla(\widetilde{U}-U)\|^2_{L_{2(\Omega)}} - \left|(a_i(\mathrm{x},\mathrm{t},\widetilde{U},\nabla U)-a_i(\mathrm{x},\mathrm{t},U,\nabla U),(\widetilde{U}-U)_{x_i})_\Omega\right|$$

Because,

$$\left|(a_i(\mathrm{x},\mathrm{t},\widetilde{U},\nabla U)-a_i(\mathrm{x},\mathrm{t},U,\nabla U),(\widetilde{U}-U)_{x_i})_\Omega\right| \le \varepsilon\|\nabla(\widetilde{U}-U)\|^2_{L_2(\Omega)} + C_0\|(\widetilde{U}-U)\|^2_{L_2(\Omega)}$$

$$\left|\left(f_n(t,\tilde{C}_n)-f_n(t,C_n),z_n\right)_{E_n}\right| = \left|a(\mathrm{x},\mathrm{t},\widetilde{U},\nabla U)-a(\mathrm{x},\mathrm{t},U,\nabla U),\widetilde{U}-U\ )_\Omega\right|$$
$$\le \varepsilon\|\nabla(\widetilde{U}-U)\|^2_{L_2(\Omega)} + C_1\|(\widetilde{U}-U)\|^2_{L_2(\Omega)}$$

Likewise,

$$\left(g_n(t,\tilde{C}_n)-g_n(t,C_n),z_n\right)_{E_n} \le \varepsilon\|\nabla(\widetilde{U}-U)\|^2_{L_2(\Omega)} + C_2\|(\widetilde{U}-U)\|^2_{L_2(\Omega)}$$

where the constants C_0, C_1 , C_2 depend on the quantities $\|U\|^2_{\mathrm{H}^1(\Omega)}$, $\|\widetilde{U}\|^2_{\mathrm{H}^1(\Omega)}$.

Then, estimating the terms of the right side of equality (16) using the Cauchy inequality and substituting the obtained estimates, setting $\varepsilon = v/6$, we obtain

$$\frac{d}{dt}\left((Q_n+\Gamma_n)z_n,z_n\right)_{E_n} + v\|\nabla(\widetilde{U}-U)\|^2_{L_2(\Omega)} \le \mathrm{C}\|\widetilde{U}-U\|^2_{\hat{L}_2} + \|\Phi_n\|^2_{E_n}$$

Integrating the last inequality from zero to t and taking into account the inequality

$$\|z_n\|^2_{E_n} \le \frac{1}{q}\|\widetilde{U}-U\|^2_{\hat{L}_2}$$

$$\left((Q_n+\Gamma_n)z_n,z_n\right)_{E_n} \ge (\mathrm{Q_n z_n,z_n})_{\mathrm{E_n}} - \alpha q\|\mathrm{z_n}\|^2_{\mathrm{E_n}} \ge (1-\alpha)(\mathrm{Q_n z_n,z_n})_{\mathrm{E_n}} = (1-\alpha)\|\widetilde{\mathrm{U}}-\mathrm{U}\|^2_{\hat{L}_2}, \quad (17)$$

$$\left|\left((Q_n+\Gamma_n)z_n(0),z_n(0)\right)_{E_n}\right| \le \left(Q_n z_n(0),z_n(0)\right)_{E_n} + \alpha q\|z_n\|^2_{E_n} \le\le c(1+\alpha)\|\widetilde{U}(x,0)-U(x,0)\|^2_{H^1(\Omega)}$$

$$\int_0^t\|\Phi_n\|^2_{E_n}dt \le 2k\|\Gamma_n\|^2 + 2K\|\delta_n(t,C_n)\|^2_{L_2(0,t,E_n)} \quad (18)$$

we get

$$\|\widetilde{U}-U\|^2_{\hat{L}_2} + \frac{v}{1-\alpha}\|\nabla(\widetilde{U}-U)\|^2_{L_2(\Omega)} \le \frac{\mathrm{C}}{1-\alpha}\int_0^t\|\widetilde{U}\ (x,t)-U(x,t)\|^2_{L_2}dt + \frac{1}{1-\alpha}(2\mathrm{k}\|\Gamma_n\|^2 +$$
$$+2\ \ \|\delta_n(t,C_n)\|_{L_2(0,t,E_n)} + c(1+\alpha)\|\widetilde{U}\ (x,0)-U(x,0)\|^2_{\hat{L}_2})$$

Denote

$$\int_0^t\|\widetilde{U}\ (x,t)-U(x,t)\|^2_{L_2}dt = y_n(t),$$

$$\frac{1}{1-\alpha}(c(1+\alpha)\|\widetilde{U}\ (x,0)-U(x,0)\|^2_{H^1(\Omega)} + \quad (19)$$
$$+2k\|\Gamma_n\|^2 + 2\max_{\|C_n\|\le K}\|\delta_n(t,C_n)\|^2_{L_2(0,t,E_n)}) = F_n(t)$$

Then, using the obtained inequalities and the lemma on differential inequalities, we obtain the inequality for $y_n(t)$ [16]

$y_n(t) \le (e^{C_1 t} - 1)/C_1 F_n(t)$ (20)

From here,

$$\left\|\widetilde{U}\ (x,t) - U(x,t)\right\|^2_{L_\infty(0,T,\hat{L}_2)} + \left\|\nabla\left(\widetilde{U} - U\right)\right\|^2_{L_2(0,t,L_2\Omega))}$$

$$\le \bar{p}_0\left\|\Gamma^0_n\ \right\|^2 + \bar{p}_1\ \|\Delta_n\|^2_{E_n} + \bar{p}_2\|\Gamma_n\|^2 + \bar{p}_3\ \|\delta_n(t, C_n)\|^2_{L_2(0,T,E_n)}$$

where constants $\overline{p}_i$ $(i = \overline{0,3})$ do not depend on n .

Conclusion. The stability of the approximate solution of the Galerkin method in the space $L_2\big(0, \mathrm{T}, \mathrm{H}^1(\Omega)\big)$ for problem (1) is established under the condition that (3)-(7) is satisfied and the coordinate system in the space $\hat{L}_2(\Omega)$ is strongly minimal.

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