

Algorithms for Selecting the Contour Lines of Images Based on the Theory of Fuzzy Sets

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Abstract: The article considers the development of algorithms and software for solving the problem of image quality assessment by fuzzy logic methods, the selection of object contours in the image and its segmentation using the fuzzy sets apparatus. The selection of the contours of an object in the image and its segmentation using the fuzzy sets apparatus.

Keywords: digital image processing, fuzzy sets, fuzzy rules, image quality improvement, object contour selection, image segmentation.

Introduction: Modern information processing systems are characterized by a high level of use of various algorithms and technologies for the intellectualization of data processing processes. One of the important directions in the field of information technology intellectualization is the processing of information in the form of photo and video data, in particular digital images.

Technologies for the intellectualization of image processing processes that are in demand when creating video surveillance and access control systems. The image processing process consists of a number of procedures, among which one of the most important is the procedures for segmentation of objects in the recorded images. Details having a relatively simple shape are uniquely characterized by the contour of objects in the images.

The tasks associated with the selection of contours of a particular object in the images are diverse and they all have their own characteristics. Therefore, there is no possibility of successful use of a single effective algorithm oriented for the selection of contour lines of objects based on digital image processing.

Literature review: Currently, a number of algorithms have been developed that are oriented to highlight the contour of objects in images [1-8]. The use of one or another algorithm when selecting contours is carried out taking into account the characteristics of the source image, the speed of its processing and from the point of view of achieving the required level of quality.

Contour highlighting is a very useful low-level image processing tool for analyzing images in the field of view of a computer and pattern recognition. In the image, the edges contain important information about the object of interest to the image, since they separate different areas of the image. Therefore, contour selection is an important part of many object recognition systems defined as images. The methods used in such recognition systems presuppose the existence of a set of features extracted based on the analysis and processing of the images in question. Feature extraction and the formation of an object image description based on digital image processing is one of the most important stages in solving applied problems using an image recognition and analysis system [2].

Undoubtedly, the most useful information in many tasks of identifying features of objects in a digital image is information about the contours of the image (that is, about lines passing on the borders of homogeneous regions). In addition, data on contour lines of images is very useful for object recognition systems represented as images. Currently, the following three approaches are mainly used to solve the problem of selecting the contours of objects in images: 1) an approach based on the calculation of discrete derivatives;

2) an approach based on statistical analysis of the brightness in the vicinity of each pixel of the image;

3) an approach based on the use of the apparatus of fuzzy set theory.

Next, we will briefly consider each approach of contour selection separately of objects in the images.

An approach based on the calculation of discrete derivatives. All the methods that are developed within the framework of this approach are based on the calculation of the first-order and second-order derivatives, and consists of two groups. The first group of methods relies on gradient operators, and the second group relies on Laplacian operators.

The main idea of gradient contour selection methods is to search for a set of pixels corresponding to essential (maximum) changes in the discrete differential of the first order. In this case, the contours of objects are determined based on the search for the maximum modulus of gradient vectors [3-6].

Contour selection using the Laplacian reduces to the definition of a discrete derivative of the second order. The main idea of the methods based on the Laplacian operators is to emphasize the discontinuities of brightness levels in the image and to suppress the region with a weak change in brightness. The main advantage of methods based on Laplacian operators is that they work very fast. The disadvantages of these procedures is that they are sensitive to noise in the image. Therefore, contour selection methods based on Laplacian operators are practically not used in solving applied problems.

Specific linear time invariant (LTI) filters are the most common procedure applied to the problem of contour selection, and the one that leads to the least computational effort. In the case of first-order filters, an edge is interpreted as a sharp change in the gray level between two adjacent pixels. The goal in this case is to determine at which points of the image the first derivative of the gray level as a function of position has a large value. By applying a threshold to the new output image, edges in arbitrary directions are detected. Early edge detection methods, such as the Sobel and Prewitt detectors, were based on the concept of spatial derivative filtering, where local gradient operators are used only to highlight contours of certain orientations. Derived filters suffer when the boundaries are blurred and noisy and are not flexible. Marr and Hildreth [7] we have proposed an algorithm that finds edges at zero intersection of the Laplacian of the image. Kenny [8] proposed a method to counteract noise problems from gradient operators, where the image is collapsed with first-order derivatives of a Gaussian filter for smoothing in the local gradient direction, followed by the selection of contours by a threshold value.

Most contour extraction algorithms apply a first and second order local derivative operator followed by some noise removal method to reduce noise. Hou and Kuo [9] used simple arithmetic and logical operations to find edges in the image. Jiang and Bank [10] presented a new method for approximating the sweep line of the edge detection line. Their algorithm has shown excellent segmentation quality and computational costs for many region-based algorithms. In [11], it calculates the difference between the center pixel and the surrounding pixel and uses the highest value among them to detect edges. In [12], I used a 5x5 window to effectively reduce noise without increasing the width of the edges, which always happen when using a 5x5 window. Since this detector uses a self-adjusting threshold, therefore it is capable of detecting edges in an area of variable grayscale.

It should be noted that gradient contour selection methods have certain advantages (in terms of sensitivity) compared to methods based on Laplacian calculations. Although these methods work best on real images compared to the method based on Laplacian calculations, they cannot provide the required accuracy when processing images that have low quality. Under these conditions, statistical methods can be used to highlight the contours of objects on images.

An approach based on the use of the apparatus of fuzzy set theory. In the last few years, fuzzy set theory has emerged as another, but powerful, tool for decision-making [13-15]. In 1965, Zadeh proposed the concept of fuzzy sets, and soon the fuzzy concept has gained popularity in the field of image processing. Many methods have been proposed by the researcher for edge detection based on the apparatus of fuzzy set theory [16-17]. In conclusion of this brief image, we note that significant advances have been made in the field of digital image processing in recent years. For example, the G.Scharr operator [17] is proposed, which are optimal among contour selection operators based on the gradient method. The methods based on the apparatus of the theory of fuzzy sets and fuzzy logic are being developed. Despite the successes achieved in the field of digital image processing, there are a number of unsolved problems. These include the problem of adequate mapping of the subject area to a fuzzy system, the choice of fuzzy inference models and their integration into a single intelligent system. At the same time, many of the developed methods based on the theory of fuzzy sets require large computational resources, which makes it difficult to apply them in application systems, for example, in biometric access control systems. Thus, the issues of digital image processing using the fuzzy sets

apparatus have not been sufficiently investigated. Therefore, the development and improvement of digital image processing methods based on the theory of fuzzy sets are very relevant.

Research methodology and empirical analysis

Statement of the contour selection problem. It is known [1-12] that, ideally, every image consists of homogeneous regions of different brightness (or, at least, allow such a representation with some error). For example, in the simplest case, the image is a set of two-color areas. At the same time, it will contain samples that can take two brightness values. Of these, only one of the brightness values correspond to objects, and the others correspond to the background. It is known that the most informative from the point of view of object recognition in images are not the brightness values of objects, but the characteristics of their boundaries – contours [4-7]. Because the main information is not in the brightness (or color) of individual areas, but in their outlines. The task of contour selection is to build an image of the boundaries of objects and the outlines of homogeneous areas.

Contour selection tasks require the use of operations performed on neighboring pixels of the image element in question. These operations, usually performed using a specific operator (for example, a gradient operator), are sensitive to changes in the brightness value of neighboring pixels. The responses of any gradient operator will be large enough on neighboring image elements when the brightness value on these pixels is significantly different. In areas of images with a constant brightness level, the responses of these operators are zero.

Consider the set of valid images I given in the color space RGB, defined by three basic colors: red (R), green (G) and a pigeon (B). At the same time, each valid image J presented in the form of a three-dimensional matrix X size $c \times m \times n$ (where - c number of color channels; m and n - the number of rows and the number of columns, respectively): $X = \parallel x_{ijk} \parallel_{m \times n}$,

where x_{ijk} - the brightness level of the element x_{ij} according to the base color.

The task of selecting the contours of objects in the image is to find a curve on $\parallel x_{ij} \parallel_{m \times n}$, bounding simply connected areas of a certain color.

When developing algorithms for contour selection, it is necessary to take into account the specified features of the behavior of contour lines. Special additional processing of selected contours allows you to eliminate gaps and suppress false contour lines. Consider the set m images of car numbers \tilde{J}^m , given in the form of a two-dimensional matrix X size $H \times W$ (где H, W - the number of rows and columns, respectively) [1]:

$$X = \begin{bmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1W} \\ \dots & \dots & \dots & \dots & \dots \\ x_{i1} & \dots & x_{ij} & \dots & x_{iW} \\ \dots & \dots & \dots & \dots & \dots \\ x_{H1} & \dots & x_{Hj} & \dots & x_{HW} \end{bmatrix}, \quad (1)$$

where x_{ij} – intensity of a given digital image ($x_{ij} \in [0, 1, 2, \dots, 255]$).

The task is to construct such a matrix X size $H \times W$ the elements of which are an indicator of contour lines:

$$X = \begin{bmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1W} \\ \dots & \dots & \dots & \dots & \dots \\ x_{i1} & \dots & x_{ij} & \dots & x_{iW} \\ \dots & \dots & \dots & \dots & \dots \\ x_{H1} & \dots & x_{Hj} & \dots & x_{HW} \end{bmatrix},$$

where x_{ij} – characterizes the element of images that is on i – om line and j – om column:

$$x_{ij} = \begin{cases} 1, & \text{if the pixel } x_{ij} \text{ is an element of contour lines;} \\ 0, & \text{otherwise.} \end{cases}$$

In this case, it is required that the elements of the matrix X the shapes of objects presented in the form of digital images have been sufficiently characterized (1).

Contour selection method

The main idea of the proposed approach is to form a contour image based on the analysis of fuzzy

increments for each pixel. In this case, the analysis is carried out by the segmentation method. Let's imagine the image in question as a set of points consisting of $W \times H$. The proposed algorithm consists of the following steps:

1 Setting the neighborhood of the central element and its increment in a certain direction.

For each central element at this stage, a sliding window is determined by the size $W_0 \times H_0$ (usually $W_0 = 2k_w + 1, H_0 = 2k_H + 1, k_w, k_H$ - natural numbers). The size of the window data is set by the parameters k_H и k_w which are determined based on the specifics of the considered problem of selecting the contours of the object in the image. After determining these parameters of the sliding window, a simple increment of the central element in question is calculated (u, v) :

$$\Delta((u, v)) = J(u + \delta_u, v + \delta_v) - J(u, v), \quad (2)$$

$$\delta_u \in \{-k_w, \dots, 0, \dots, k_w\}, \delta_v \in \{-k_H, \dots, 0, \dots, k_H\}, |\delta_u| + |\delta_v| \neq 0,$$

where δ_u, δ_v - integers.

In order to simplify the further presentation of the meaning δ_u и δ_v it is assumed that $\delta_u = \delta_v = 1$. Due to the limited scope of this work, the main arguments at subsequent stages are given only for $\delta_u = \delta_v = 1$. Then for each central element (u, v) eight simple increments can be calculated using the formula (3):

To define large (or small) fuzzy increments, we introduce the concept of a set of fuzzy ones among the calculated increments. It is known [15-20] that the set of fuzzy increments is determined using the membership function of its elements.

2 Construction of the membership function for a set of fuzzy increments. At this stage, the membership function of fuzzy increments provides:

- 1) adaptations to noise components when performing fuzzy smoothing;
- 2) the difference between noise and structural image objects.

It is assumed that the noise is distributed evenly throughout the images. The main task of this stage is to determine the fuzzy increments by the values of the surrounding neighboring pixels of the central pixel. To solve this problem, estimates are calculated that characterize the increments of each central element in a certain direction. The construction of the membership function is based on a simple heuristic fuzzy increment, according to this heuristic increment, corresponds to the boundaries of objects, and a small fuzzy increment corresponds to noise. It is known [15-22] that constructing the membership function in the form of some simple mathematical function is the most convenient, which simplifies the corresponding computational and reduces computational resources. When constructing membership functions, the method of parametric representation is used, which provides the prostate of construction. The choice of the membership function depends on the problem. In this algorithm, triangular membership functions are used for both input and output. The standard triangular membership function is defined by the formula (3):

$$\mu(x; a, b, c) = \begin{cases} 0, & \text{при } x \leq a; \\ \frac{x-a}{b-a}, & a \leq x \leq b; \\ \frac{c-x}{c-b}, & b \leq x \leq c; \\ 0, & x \geq c. \end{cases} \quad (3)$$

Here a, b, c - some numeric parameters that take integer values: $a \leq b \leq c$ и $a, b, c \in \{0, 1, 2, \dots, k\}$, where x - the number of gradations on the processed images.

Parameters a, c - defines the base of the triangle, and the a parameter b - its top. It should be noted that the reduced membership function generates a normal convex and unimodal fuzzy set with a carrier interval (a, c) , the core $\{b\}$ and fashion b .

In order to clarify the concept of fuzzy increments, we introduce the qualitative concept of "small". Within the framework of fuzzy set theory. This concept corresponds to fuzzy sets of small numbers can be reduced, т.е. $a = -b, b = 0, c = k_0$.

$$\mu_k(x, k_b) = \begin{cases} b, & \text{by } \in [-k_a, \dots, k_0]; \\ 0, & \text{by } \notin [-k_a, \dots, k_0] \end{cases} \quad (4)$$

Here $b = (1 - |x|/k_a)$ where k_a - an adaptation parameter that characterizes a particular set of small numbers. The membership functions of the qualitative concepts of large increment can be determined based on the formula (4):

$$\mu_k(x) = 1 - \mu_L(x, k_a).$$

After constructing the fuzzy increment membership function, it becomes possible to calculate the values of fuzzy increments in all directions.

2 Calculating the values of fuzzy increments at this stage, a set of fuzzy rules is set. It is known [15-22] that fuzzy rules are constructed from the membership functions of the input and output images. These rules define the relationship between the premise and the conclusion expressed in the form of “if ... then...”. Gasified inputs and outputs make it much easier to isolate rules and generalize them. Building good rules depends on the amount and quality of the expert's knowledge of image processing.

However, the description of the experts' knowledge is formalized. Therefore, the choice, for example, of the form and parameters of the membership function is arbitrary. When calculating the fuzzy increment value for an arbitrary central element in each direction, the corresponding fuzzy rules are used. In cases where the increment is already defined in eight directions, for example, it is set to calculate a fuzzy increment in the first direction, i.e. $d_1(u, v) = J(u - 1, v - 1) - J(u, 0)$;

To calculate the values of fuzzy increments for a pixel (u, v) in the direction *NW* you can use the following rules:

If $(\mathcal{P}_1(u, v) = 1)$ to $\Delta_1(u, v)$ small, where $\mathcal{P}_1(u, v)$ – predicate describing (characterizing) increments. Calculating the value of the predicate $\mathcal{P}(u, v)$ this rule is implemented according to the formula (5):

$$\mathcal{P}_1(u, v) = P_1^1(u, v) \vee P_1^2(u, v) \vee P_1^3(u, v), \quad (5)$$

$$P_1^1(u, v) = P_1(u, v) \wedge P_1(u - 1, v + 1),$$

$$P_1^2(u, v) = P_1(u, v) \wedge P_1(u + 1, v - 1),$$

$$P_1^3(u, v) = P_1(u - 1, v + 1) \wedge P_1(u + 1, v - 1),$$

where
$$P_1(u, v) = \begin{cases} 1, & \text{if } d_1(u, v) \text{ small;} \\ 0, & \text{otherwise.} \end{cases}$$

To calculate the increment in all directions, all the rules are used $\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_8$.

3 Formation of a set of fuzzy features. At this stage, a number of fuzzy characteristics are defined to describe each acceptable element \mathfrak{B} the original image. To transform the elements of the source image in a set of feature vectors, the following fuzzy characteristics can be distinguished for each pixel of the analyzed source image:

1 Fuzzy pixel increments \mathfrak{p}_i on coordinates \mathbf{x}, \mathbf{y} for each of the three basic colors ($i = 1, 2, 3$) calculated at the second stage:

$$\mathfrak{p}_i = \{z | \mu_{C_i}(z) \geq 0, z \in \mathfrak{X}, \\ z = J_i(x, y), \mathfrak{X} = \{0, 1, \dots, 255\}$$

2 Fuzzy brightness \mathfrak{S}_i an arbitrary fragment of an image consisting of 3×3 elements ($i = 1, 2, 3$):

$$\mathfrak{S}_i = \{s_i | \mu_{C_i}(s_i) \geq 0, s_i \in \mathfrak{S}, \mathfrak{S} \subseteq \mathfrak{X},$$

Where s_i – average brightness i - the color in the neighborhood of the pixel with coordinates (x, y) :

$$s_i = \left(\sum_{u=-1}^1 \sum_{v=-1}^1 J_i(x + u, y + v) \right) / 9.$$

To determine the fuzzy brightness (when forming averaged brightness features on an image fragment) in the vicinity of a pixel with coordinates (\mathbf{x}, \mathbf{y}) .

3 Fuzzy estimation of the average quadratic brightness value for an image fragment consisting of 3×3 elements ($i = 1, 2, 3$):

$$\mathfrak{E}_i = \{e_i | \mu_{C_i}(e_i) \geq 0, e_i \in \mathfrak{D}, \mathfrak{D} \subseteq \mathfrak{X},$$

Where e_i – average brightness i - the color of the pixel with coordinates (x, y) :

$$e_i = \sum_{u=-1}^1 \sum_{v=-1}^1 (J_i(x + u, y + v))^2.$$

4 Fuzzy estimation of brightness dispersion over an image fragment consisting of 3×3 элементов ($i = 1, 2, 3$):

$$\mathfrak{D}_i = \{d_i | \mu_{C_i}(d_i) \geq 0, d_i \in \mathfrak{D}, \mathfrak{D} \subseteq \mathfrak{X},$$

Where d_i – average brightness i - the color of the pixel with coordinates (x, y) :

$$\delta_i = \left(\sum_{u=-1}^1 \sum_{v=-1}^1 (f_i(u, v))^2 \right) / 9,$$

$$f_i(u, v) = J_i(x + u, y + v) - s_i(x, y).$$

Where $s_i(x, y)$ – average brightness i - th color in the neighborhood of the pixel with coordinates (x, y) .

4 Definition of image contours using the segmentation method. Dividing the image in question into l segments are performed based on the formation of subsets of connected pixels. The basic idea of forming subsets of strongly connected elements is that the elements of each segment will be closer to its "center" than to the "centers" of other segments. The problem of forming subsets of strongly connected elements is considered solved if by the set \mathfrak{R} it was possible to determine the "centers" of the segments and the boundaries of the corresponding subsets of elements. The proximity to the central element of the segments is determined based on the concept of a fuzzy set [13,15].

Consider the set of valid elements (i.e., valid pixels) of an image \mathfrak{R} . Suppose that each valid element p ($p \in \mathfrak{R}$) corresponds to the feature vector $\bar{a} = (a_1, \dots, a_i, \dots, a_n)$, calculated at the second stage.

The algorithm of fuzzy segmentation of image elements can be described as follows. Let the parameters be given l, C_j ($j = \overline{2, l}$), which characterize the number of segments and segment centers, respectively. Then to determine the original partition $\mathcal{R}_0(\mathfrak{R}) = \{S_j, |S_j \subset \mathfrak{R}\}$ it is possible to calculate the centers of the segments by formula (2.2) and the value of the objective function by formula (2.3). If for some j ($j \in \{2, \dots, l\}$) and some r_{xy} ($r_{xy} \in \mathfrak{R}$) meaning

$$\sum_{i=1}^N (r_{xy}^i - v_{ij}(x, y)) = 0,$$

then for the corresponding fuzzy segment S_j it is believed that $\mu_j(r_{xy}) = 1$, a for segments S_q ($q = \overline{1, l}, j \neq q$) it is believed that $\mu_j(r_{xy}) = 0$.

Further, for the obtained fuzzy segments, the refinement of the "centers" of the segments and the value of the objective function, respectively, according to the formula (2) and (3).

The calculation of the "centers" of the segments is carried out on the basis of an iterative method that relies on sequential refinement of the value z_{ij} (see formula 2) in each iteration. At the same time, the value $\mu_{S_j}(J_u)$ calculated as follows:

$$\mu_{S_j}(J_u) = \delta_{uj} / \left(\sum_{v=1}^l \delta_{uv} \right),$$

$$\delta_{uj} = 1 / \left(\sum_{i=1}^n (z_{ij} - \tau_{iu}) \right),$$

$$j \in \{1, 2, \dots, l\}, u \in \{1, 2, \dots, m\}.$$

During the implementation of the fuzzy segmentation algorithm, the process of clarifying the "centers" of segments is performed. This process is terminated if:

- 1) the condition is met $|\mathcal{D}(\mathcal{R}_{q-1}) - \mathcal{D}(\mathcal{R}_q)| \leq \varepsilon$;
- 2) the number of completed iterations q exceeds the specified number q_0 .

Experimental studies were carried out to verify the operability of the considered algorithm. The proposed algorithm for selecting the contours of an object in the image was modeled using MATLAB. The performance of the proposed method is compared with the Sobel and Prewitt operators. As a result, it was noted that the proposed contour extraction algorithm is better than the Sobel and Prewitt operators in finding different edges from images and, thus, can provide a better display of edges, which is impossible in the case of the Sobel and Prewitt operator.

Results. The considered approach to improving color images is based on the method of stretching the dynamic range. However, excessive amplification of intensity values does not always lead to the desired results, especially when it comes to color images. Therefore, adjustable amplification of pixel intensity values will be

implemented in this method. It should also be noted that an important parameter when improving images is the size of the local neighborhood. It significantly affects the detail of the resulting image.

Here is an example of a software implementation of the method.



First, the original image is read into the workspace.

Figure 1 - The original image

Then, for each of the color components, the parameters of local neighborhoods are determined – minimum, maximum and standard deviation of pixel intensities. All these parameters are defined for a local neighborhood with dimensions r .

The corresponding gain is also set: K_r ; K_g ; K_b ; and the values of the intensities of the color components of the resulting image are determined.

We present the results obtained for different values of the sizes of local neighborhoods and different values of the gain.



Figure 2 - Size of the local neighborhood 3×3 , gain $K_r=K_g=K_b=3$



Figure 3 - Size of the local neighborhood 3×3 , gain $K_r=K_g=K_b=7$



Figure 4 - Size of the local neighborhood 3×3 , gain $K_r=K_g=K_b=15$



Figure 5 - Size of the local neighborhood 7 x 7, gain $K_r=K_g=K_b=3$



Figure 6 - Size of the local neighborhood 7 x 7, gain $K_r=K_g=K_b=7$



Figure 7 - Size of the local neighborhood 7 x 7, gain $K_r=K_g=K_b=15$



Figure 8 - Size of the local neighborhood 15 x 15, gain $K_r=K_g=K_b=3$



Figure 9 - Size of the local neighborhood 15 x 15, gain $K_r=K_g=K_b=7$



Figure 10 - The size of the local neighborhood 15 x 15, gain $K_r=K_g=K_b=15$

It can be seen from the presented results that the method described above is an effective means of improving color images. Using different sizes of local neighborhoods and coefficient values K , you can adjust the level of image detail and pixel intensity.

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