

We multiply the first row by $c_{21} = -\frac{a_{21}}{a_{11}}$ and add it to the second row of the matrix, we multiply the first

row by $c_{i1} = -\frac{a_{i1}}{a_{11}}$ and add it to the i -row, as a result the following matrix is formed:

$$\begin{pmatrix} a_{11}^{(1)} & a_{12}^{(1)} & \dots & a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & \dots & a_{2n}^{(2)} \\ \dots & \dots & \ddots & \dots \\ 0 & a_{n2}^{(2)} & \dots & a_{nn}^{(2)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1^{(1)} \\ b_2^{(2)} \\ \vdots \\ b_n^{(2)} \end{pmatrix} \quad (3)$$

Here $a_{ij}^{(2)} = a_{ij}^{(1)} + c_{i1}a_{1j}^{(1)}$, $b_i^{(2)} = b_i^{(1)} + c_{i1}b_1^{(1)}$, $i \geq 2$.

➤ **The second step:**

We multiply the second row by $c_{32} = -\frac{a_{32}^{(2)}}{a_{22}^{(2)}}$ and add it to the third row of the matrix, if we multiply the

second row by $c_{i2} = -\frac{a_{i2}^{(2)}}{a_{22}^{(2)}}$ and add it to the i -row of the matrix for the condition $i > 2$:

$$\begin{pmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & \dots & a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & \dots & a_{2n}^{(2)} \\ 0 & 0 & a_{33}^{(3)} & \dots & a_{3n}^{(3)} \\ \dots & \dots & \dots & \ddots & \dots \\ 0 & 0 & a_{n3}^{(3)} & \dots & a_{nn}^{(3)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1^{(1)} \\ b_2^{(2)} \\ b_3^{(3)} \\ \vdots \\ b_n^{(3)} \end{pmatrix} \quad (4)$$

and finally, in step $k+1$ - constant coefficients are $a_{ij}^{(k+1)} = a_{ij}^{(k+1)} + c_{ik}a_{kj}^{(k)}$, free terms

$b_i^{(k+1)} = b_i^{(k)} + c_{ik}b_k^{(k)}$, where $c_{ik} = -\frac{a_{ik}^{(k)}}{a_{kk}^{(k)}}$, $i, j > k$.

➤ **$n-1$ -step:**

$$\begin{pmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & \dots & a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & \dots & a_{2n}^{(2)} \\ 0 & 0 & a_{33}^{(3)} & a_{34}^{(3)} & \dots & a_{3n}^{(3)} \\ 0 & 0 & 0 & a_{44}^{(4)} & \dots & a_{4n}^{(4)} \\ \dots & \dots & \dots & \dots & \ddots & \dots \\ 0 & 0 & 0 & 0 & 0 & a_{nn}^{(n)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1^{(1)} \\ b_2^{(2)} \\ b_3^{(3)} \\ b_4^{(4)} \\ \vdots \\ b_n^{(n)} \end{pmatrix} \quad (5)$$

So, in the system of equations, the unknown value of n - is equal to $x_n = \frac{b_n^{(n)}}{a_{nn}^{(n)}}$. In general, it will be

$A_{nn}x_n = b_{nn}$. Let's define $A_{nn} = U$, $b_{nn} = f$. As a result, the general formula for the solution of the system of linear algebraic equations

$$x_k = \frac{1}{U_{kk}} \left(f_k - \sum_{i=k+1}^n U_{ki}x_i \right), \quad i = n, n-1, n-2, \dots, 1 \quad (6)$$

is derived.

An example. Let it be required to solve this system of linear algebraic equations:

$$\begin{cases} x + 2y + z = 4 \\ 3x - 5y + 3z = 1 \\ 2x + 7y - z = 8 \end{cases}$$

The corresponding matrix representation of the given equation is written as follows:

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & -5 & 3 \\ 2 & 7 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix}$$

We calculate invariant coefficients and free terms using the formulas and condition of matrix elements $a_{ij}^{(k+1)} = a_{ij}^{(k)} + c_{ik} a_{kj}^{(k)}$, $b_i^{(k+1)} = b_i^{(k)} + c_{ik} b_k^{(k)}$, $c_{ik} = -\frac{a_{ik}^{(k)}}{a_{kk}^{(k)}}$, $i, j > k$. It is considered that the given system of linear algebraic equations consists of three equations with three unknowns. In the first step, the values of the proportionality coefficients c_{21} , c_{31} , c_{32} are calculated, then the values of the coefficients and free terms when $k = 1$ and $k = 2$ are determined.

Values of coefficients for case $k = 1$:

$$c_{21} = -\frac{a_{21}^{(1)}}{a_{11}^{(1)}} = -\frac{3}{1} = -3;$$

$$a_{21}^{(2)} = a_{21}^{(1)} + c_{21} a_{11}^{(1)} = 3 + (-3) \cdot 1 = 0;$$

$$a_{22}^{(2)} = a_{22}^{(1)} + c_{21} a_{12}^{(1)} = -5 + (-3) \cdot 2 = -11;$$

$$a_{23}^{(2)} = a_{23}^{(1)} + c_{21} a_{13}^{(1)} = 3 + (-3) \cdot 1 = 0;$$

$$c_{31} = -\frac{a_{31}^{(1)}}{a_{11}^{(1)}} = -\frac{2}{1} = -2;$$

$$a_{31}^{(2)} = a_{31}^{(1)} + c_{31} a_{11}^{(1)} = 2 + (-2) \cdot 1 = 0;$$

$$a_{32}^{(2)} = a_{32}^{(1)} + c_{31} a_{12}^{(1)} = 7 + (-2) \cdot 2 = 3;$$

$$a_{33}^{(2)} = a_{33}^{(1)} + c_{31} a_{13}^{(1)} = -1 + (-2) \cdot 1 = -3;$$

Values of coefficients for case $k = 2$:

$$c_{32} = -\frac{a_{32}^{(2)}}{a_{22}^{(2)}} = -\frac{3}{11};$$

$$a_{31}^{(3)} = a_{31}^{(2)} + c_{32} a_{21}^{(2)} = 0 + (3/11) \cdot 0 = 0;$$

$$a_{32}^{(3)} = a_{32}^{(2)} + c_{32} a_{22}^{(2)} = 3 + (3/11) \cdot (-11) = 0;$$

$$a_{33}^{(3)} = a_{33}^{(2)} + c_{32} a_{23}^{(2)} = -3 + (3/11) \cdot 0 = -3;$$

Values of free terms:

$$b_1^{(1)} = 4;$$

$$b_2^{(2)} = b_2^{(1)} + c_{21} b_1^{(1)} = 1 + (-3) \cdot 4 = -11;$$

$$b_3^{(2)} = b_3^{(1)} + c_{31} b_1^{(1)} = 8 + (-2) \cdot 4 = 0;$$

$$b_3^{(3)} = b_3^{(2)} + c_{32} b_2^{(2)} = 0 + (3/11) \cdot (-11) = -3;$$

The resulting matrix takes the following form:

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -11 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -11 \\ -3 \end{pmatrix}$$

Using the matrix multiplication property, the following system of equations is derived:

$$\begin{cases} x + 2y + z = 4 \\ -11y = -11 \\ -3z = -3 \end{cases}$$

The answer: $x = y = z = 1$.

Above we found the solution of the system of equations using the numerical method, which is a bit more difficult and time consuming to find the solution. Taking this into account, it is important to use programming languages to solve the system of equations.

Let's start the Html programming language: using equation (6), the program code in the Html environment is created as follows:

```
<!DOCTYPE html>
<html>
<head>
  <title>6.21</title>
  <meta charset="utf-8">
</head>
<body>
  <h3> Solving the system of linear equations by the Matrix method </h3>
  <?php
echo "x" . "+" . "y" . "+" . "z" . "=4" ;
echo "<br>";
echo "x" . "+" . "2y" . "+" . "4z" . "=4" ;
echo "<br>";
echo "x" . "+" . "3y" . "+" . "9z" . "=2";
echo "<br>";
for($i=0;$i<3;$i++){
  echo "<br>" . "&nbsp;";
}
$a11=0;
$a12=0;
$a13=0;
$a21=0;
$a22=0;
$a23=0;
$a31=0;
$a32=0;
$a33=0;
$a=0;
$b=0;
$s=0;
if(isset($_POST["a12"]) && isset($_POST["a12"]) && isset($_POST["a13"]) &&
isset($_POST["a21"]) && isset($_POST["a22"]) && isset($_POST["a23"]) && isset($_POST["a31"]) &&
isset($_POST["a32"]) && isset($_POST["a33"]) && isset($_POST["a"]) && isset($_POST["b"]) &&
isset($_POST["s"])){
  $a11=$_POST["a11"];
  $a12=$_POST["a12"];
```

```

    $a13=$_POST["a13"];
    $a21=$_POST["a21"];
    $a22=$_POST["a22"];
    $a23=$_POST["a23"];
    $a31=$_POST["a31"];
    $a32=$_POST["a32"];
    $a33=$_POST["a33"];
    $a=$_POST["a"];
    $b=$_POST["b"];
    $s=$_POST["s"];
}
$S=$a11*$a22*$a33+$a12*$a23*$a31+$a13*$a21*$a32-$a13*$a22*$a31-$a11*$a23*$a32-$a12*$a21*$a33;
echo "delta" . "=" . $S . "<br>";
$a1=$a22*$a33-$a23*$a32;
$a2=-($a21*$a33-$a23*$a31);
$a3=$a21*$a32-$a22*$a31;
$b1=-($a12*$a33-$a13*$a32);
$b2=$a11*$a33-$a13*$a31;
$b3=-($a11*$a32-$a12*$a31);
$c1=$a12*$a23-$a13*$a22;
$c2=-($a11*$a23-$a21*$a13);
$c3=$a11*$a22-$a12*$a21;
echo $a1 . " " . $b1 . " " . $c1 . " " . "<br>" .
    $a2 . " " . $b2 . " " . $c2 . " " . "<br>" .
    $c3 . " " . $b3 . " " . $c3 ;
for($i=0;$i<3;$i++){
    echo "<br>" . "&nbsp;";
}
$x11=$a1/$S;
$x12=$b1*1/$S;
$x13=$c1*1/$S;
$x21=$a2*1/$S;
$x22= $b2*1/$S;
$x23=$c2*1/$S;
$x31= $c3*1/$S ;
$x32=$b3*1/$S;
$x33=$c3*1/$S;
echo $x11 . " " . $x12 . " " . $x13 . " " . "<br>" .
    $x21 . " " . $x22 . " " . $x23 . " " . "<br>" .
    $x31 . " " . $x32 . " " . $x33 ;
for($i=0;$i<3;$i++){
    echo "<br>" . "&nbsp;";
}
echo $x11*$a . " " . $x12*$b . " " . $x13*$s . " " . "<br>" .
    $x21*$a . " " . $x22*$b . " " . $x23*$s . " " . "<br>" .
    $x31*$a . " " . $x32*$b . " " . $x33*$s . "<br>" ;
$q= $x11*$a+$x12*$b+$x13*$s;
$w= $x21*$a+$x22*$b+$x23*$s;
$e= $x31*$a+$x32*$b+$x33*$s;

for($i=0;$i<3;$i++){
```

```
    echo "<br>" . "&nbsp;";
}
echo "X" . "=" . $q . "<br>";
echo "Y" . "=" . $w . "<br>";

echo "Z" . "=" . $e . "<br>";
?>
<form method="POST">
    <p>a1,1:<input type="number" name="a11">a1,2:<input type="number" name="a12">a1,3:<input
type="number" name="a13"></p>
    <p>a2,1:<input type="number" name="a21">a2,2:<input type="number" name="a22">a2,3:<input
type="number" name="a23"></p>
    <p>a3,1:<input type="number" name="a31">a3,2:<input type="number" name="a32">a3,3:<input
type="number" name="a33"></p>
    <p>a son:<input type="number" name="a"></p>
    <p>b son:<input type="number" name="b"></p>
    <p>s son:<input type="number" name="s"></p>
<button type="submit">chiqarish</button>
</form>
</body>
</html>
```

Above, we learned how to solve the system of linear algebraic equations in a number of ways and found the solution in the Html environment.

In conclusion, in the "Higher Mathematics" course, students work with matrices, perform various operations on them, learn different methods of solving a system of n unknown linear equations in the Html environment, and work in the Html environment increases the knowledge and skills of the students. This gives practical results.

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